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Mateusz Klonowski and Michał Oleksowicz

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Logo of the PCL on the front page
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Contents

Keynote Speakers

Sankha S. Basu, Indraprastha Institute of Information Technology Delhi, India

Johan van Benthem, University of Amsterdam, Netherlands; Stanford University, United States of America

Otávio Bueno, University of Miami, United States of America

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Zalán Gyenis, Jagiellonian University, Poland

Andrzej Indrzejczak, University of Lodz, Poland

Stavros Ioannidis, National and Kapodistrian University of Athens, Greece

Wojciech Jamroga, Polish Academy of Sciences, Poland; Nicolaus Copernicus University in Toruń, Poland

Fengkui Ju, Beijing Normal University, China

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Tomasz Placek, Jagiellonian University, Poland

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Accompanying Events

The PCL will feature the following workshops:

- **3rd Workshop on Relating Logic** (WRL3)

  Organized by Mateusz Klonowski (Nicolaus Copernicus University in Toruń) and Jacek Malinowski (Polish Academy of Sciences).

- **1st Workshop on Mechanisms and Causes** (WMaC1)

  Organized by Michał Oleksowicz (Nicolaus Copernicus University in Toruń) and Mateusz Chwastyk (Polish Academy of Sciences).

- **1st Symposium on the Languages and Logics of Syllogistics** (SYLLOS1)

  Organized by Luis Estrada-González (National Autonomous University of Mexico) and Tomasz Jarmużek (Nicolaus Copernicus University in Toruń).

- **3rd Workshop on Non-Fregean Logics** (WNFL3)

  Organized by Dorota Leszczyńska-Jasion and Szymon Chlebowski (Adam Mickiewicz University in Poznań).

During the PCL there will also be a tutorial entitled "**Formal Theories of Definite Descriptions**" (Tutorial FTDD). The tutorial is prepared by **Nils Kürbis** and **Michał Zawidzki** (University of Lodz).

Another event that will take place during the PCL is a panel discussion entitled "**Logic in the Development of Artificial Intelligence**". The panel will be attended by:

- **Rajeev Gore**, Monash University, Australia
- **Wojciech Jamroga**, Nicolaus Copernicus University in Toruń, Poland; Polish Academy of Sciences, Poland
- **Damian Niwiński**, University of Warsaw, Poland
- **Marek Sergot**, Imperial College London, England
- **Josef Urban**, Czech Technical University in Prague, Czech Republic

General Session

Invited Lectures

Explosion Principles in Logic

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Joint work with Sayantan Roy

Tags: non-classical logic, paraconsistent logic, substructural logic.

In this work, we discuss generalizations of the well-known *principles of explosion* in logics, the failure of which lead to the logics being termed *paraconsistent*. Before we go into any further details, by a logic, we mean a pair $\langle \mathcal{L}, \vdash \rangle$, where $\mathcal{L}$ is a set and $\vdash \subseteq \mathcal{P}(\mathcal{L}) \times \mathcal{L}$. This is what is called a *logical structure* in the study of universal logic [1]. Any logic in the usual sense of the term is a special case of this, where $\mathcal{L}$ is a formula algebra generated over a set of variables V using a finite set of connectives/operators. We do not, however, dwell on this distinction here. It is clearly indicated in the discourse where a logic in the usual sense is used.

The talk can be divided into two parts, we can perhaps call these 'syntactic' and 'semantic.'

1. Syntactic explosion

We propose that a principle of explosion in a logic $\langle \mathcal{L}, \vdash \rangle$ can be described broadly as a rule that allows one to entail every $\beta \in \mathcal{L}$ from a $\Gamma \subsetneq \mathcal{L}$. Such a set Γ, such that $\Gamma \vdash \beta$ for all $\beta \in \mathcal{L}$, is then said to *explode*. One of the most common such principle of explosion is the *ex contradictione sequitur quodlibet (ECQ)*. This is usually expressed symbolically as follows. For any $\alpha \in \mathcal{L}$, $\{\alpha, \neg\alpha\} \vdash \beta$ for all $\beta \in \mathcal{L}$. A logic $\mathcal{S} = \langle \mathcal{L}, \vdash \rangle$ is then said to be *paraconsistent* if ECQ fails in it. Some more principles of explosion have been discussed in [2]. These different forms of explosion are not necessarily equivalent. It is thus, in general, possible to have different notions of paraconsistency via the failure of different principles of explosion.

It is noteworthy, however, that a common thread tying these principles of explosion is the dependence on the language via the use of negation or other operators. The principles of explosion discussed in this work explore the question of whether a principle of explosion, and hence a notion of paraconsistency, can be described without the use of connectives.

We begin with the following principle of explosion as a direct generalization of ECQ. Suppose $\mathcal{S} = \langle \mathcal{L}, \vdash \rangle$ is a logic.

- For each $\alpha \in \mathcal{L}$ there exists $\beta \in \mathcal{L}$ such that $\{\alpha, \beta\} \vdash \gamma$, where $\gamma \in \mathcal{L}$ is arbitrary. [gECQ]

The letter 'g' above stands for 'generalized'. A logic $\mathcal{S} = \langle \mathcal{L}, \vdash \rangle$ is then said to be *NF-paraconsistent* (NF stands for negation-free) if gECQ fails in it.

An important class of NF-paraconsistent logics has been obtained within the class of *variable inclusion logics* (these are logics in the usual sense) [3].

We next go on to further generalized principles of explosion. These are listed as follows. As before, $\mathcal{S} = \langle \mathcal{L}, \vdash \rangle$ is a logic in the general sense, i.e., $\mathcal{L}$ is a set and $\vdash \subseteq \mathcal{P}(\mathcal{L}) \times \mathcal{L}$. Let $C_\vdash$ be the operator on $\mathcal{P}(\mathcal{L})$ corresponding to $\vdash$, i.e., $C_\vdash : \mathcal{P}(\mathcal{L}) \to \mathcal{P}(\mathcal{L})$ defined by $C_\vdash(\Gamma) = \{\alpha \in \mathcal{L} \mid \Gamma \vdash \alpha\}$ for any $\Gamma \subseteq \mathcal{L}$.

- For each $\alpha \in \mathcal{L}$, there exists a set $\Gamma \subsetneq \mathcal{L}$ such that $\alpha \in \Gamma$ and $C_\vdash(\Gamma) = \mathcal{L}$. [sECQ]
- For each $\alpha \in \mathcal{L}$, there exists a set $\Gamma \subsetneq \mathcal{L}$ such that $\Gamma \cup \{\alpha\} \subsetneq \mathcal{L}$ and $C_\vdash(\Gamma \cup \{\alpha\}) = \mathcal{L}$. [sECQ′]

The letter ‘s’ in the above principles denotes that these are ‘set-based’ principles of explosion.

- For each $\Gamma \subsetneq \mathcal{L}$, there exists $\alpha \in \mathcal{L}$ such that $\Gamma \cup \{\alpha\} \subsetneq \mathcal{L}$ and $C_\vdash(\Gamma \cup \{\alpha\}) = \mathcal{L}$. [spECQ]

The letter-pair ‘sp’ above denotes ‘set-point’ in reference to the manner in which the explosion takes place.

- For each $\Gamma \subsetneq \mathcal{L}$, there exists $\Delta \subsetneq \mathcal{L}$ such that $\Gamma \subseteq \Delta$ and $C_\vdash(\Delta) = \mathcal{L}$. (pfECQ1)
- For each $\Gamma \subsetneq \mathcal{L}$, there exists $\Delta \subsetneq \mathcal{L}$ such that $\Gamma \cup \Delta \subsetneq \mathcal{L}$ and $C_\vdash(\Gamma \cup \Delta) = \mathcal{L}$. (pfECQ2)
- For each $\Gamma \subsetneq \mathcal{L}$, there exists $\emptyset \neq \Delta \subsetneq \mathcal{L}$ such that $\Gamma \cup \Delta \subsetneq \mathcal{L}$, and for every $\emptyset \neq \Delta' \subseteq \Delta$, $C_\vdash(\Gamma \cup \Delta') = \mathcal{L}$. (pfECQ3)

The letter-pair ‘pf’ in the above principles denotes ‘point-free’ indicating that these are entirely free from any reference to elements of $\mathcal{L}$.

The relationships between these various principles of explosion have been thoroughly investigated. The results can be summarized in Figure 1 below.

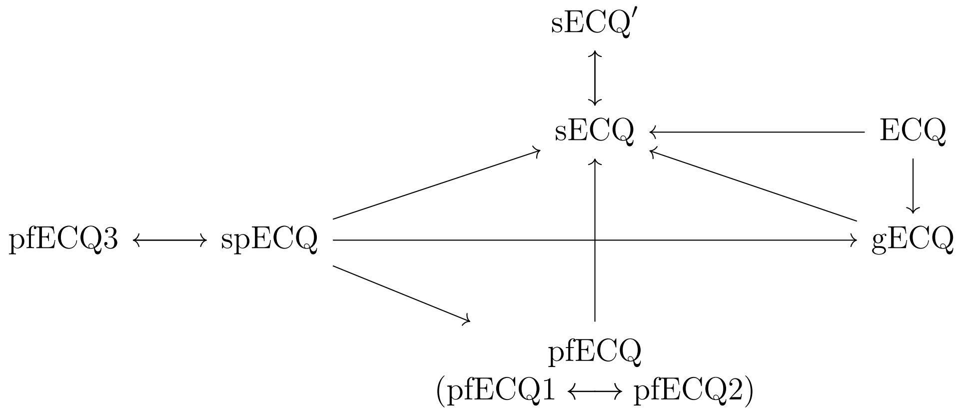

Figure 1: Principles of Explosion

2. Semantic explosion

The motivation behind studying semantic explosion separately stems from the observation that there is a difference between the syntactic and semantic approaches to explosion. Suppose $(\mathcal{L}, \vdash)$ is a syntactic presentation, say a Hilbert-style presentation, of classical propositional logic (CPL). Then, for any $\alpha, \beta \in \mathcal{L}$, there is a derivation of β from $\{\alpha, \neg\alpha\}$ ($\neg$ denotes the classical negation). Thus, we have ECQ. Now, once CPL is interpreted via valuations from $\mathcal{L}$ to the 2-element Boolean algebra, ECQ is a valid rule only because of the absence of any valuation v such that $v(\alpha) = v(\neg\alpha) = 1$. Thus, semantically speaking, explosion happens due to contradictory sets being unsatisfiable. We thus first define an abstract framework for describing semantics as follows.

An *abstract model structure* (amst) is a triple of the form $\mathfrak{M} = (\mathbf{M}, \models, \mathcal{P}(\mathcal{L}))$, where $\mathbf{M}\,(\neq \emptyset), \mathcal{L}$ are sets, and $\models \subseteq \mathbf{M} \times \mathcal{P}(\mathcal{L})$. One can think of $\mathcal{L}$ as the set of *sentences* or *well-formed formulas (wffs)* of some language, $\mathbf{M}$ as a set of *structures* (or *models*) for $\mathcal{L}$, and $\models$ as the *satisfaction relation* between sets of sentences and models.

Given an amst $\mathfrak{M} = (\mathbf{M}, \models, \mathcal{P}(\mathcal{L}))$, $m \in \mathbf{M}$, and a set $\Gamma \subseteq \mathcal{L}$, we say that m *satisfies* Γ *in* $\mathfrak{M}$ if $m \models \Gamma$. A set $\Gamma \subseteq \mathcal{L}$ is said to be *satisfiable in* $\mathfrak{M}$ if there exists $m \in \mathbf{M}$ such that m satisfies Γ, and *finitely satisfiable in* $\mathfrak{M}$ if every finite subset of Γ is satisfiable. An amst $\mathfrak{M} = (\mathbf{M}, \models, \mathcal{P}(\mathcal{L}))$ is said to be *compact* if for all $\Gamma \subseteq \mathcal{L}$, Γ is satisfiable in $\mathfrak{M}$ iff it is finitely satisfiable in $\mathfrak{M}$.

The first set of semantic explosion principles are as follows. Suppose $\mathfrak{M} = (\mathbf{M}, \models, \mathcal{P}(\mathcal{L}))$ is an amst.

- gECQ-sat holds in $\mathfrak{M}$ if, for all $\alpha \in \mathcal{L}$, there exists $\beta \in \mathcal{L}$ such that $\{\alpha, \beta\}$ is not satisfiable.
- sECQ-sat holds in $\mathfrak{M}$ if, for all $\alpha \in \mathcal{L}$, there exists $\Gamma \subseteq \mathcal{L}$ such that $\Gamma \cup \{\alpha\} \subsetneq \mathcal{L}$ and $\Gamma \cup \{\alpha\}$ is not satisfiable.
- spECQ-sat holds in $\mathfrak{M}$ if, for all $\Gamma \subsetneq \mathcal{L}$, there exists $\alpha \in \mathcal{L}$ such that $\Gamma \cup \{\alpha\} \subsetneq \mathcal{L}$ and $\Gamma \cup \{\alpha\}$ is not satisfiable.
- pfECQ-sat holds in $\mathfrak{M}$ if, for all $\Gamma \subsetneq \mathcal{L}$, there exists $\Delta \subsetneq \mathcal{L}$ such that $\Gamma \subseteq \Delta$ and Δ is not satisfiable.

The letters g, s and the letter-pairs sp, pf stand for, respectively, 'generalized,' 'set-based,' 'set-point,' and 'point-free.' The postfix sat in the names of the above principles indicate that these are formulated in terms of satisfiability.

Suppose $\mathfrak{M} = (\mathbf{M}, \models, \mathcal{P}(\mathcal{L}))$ is an amst. Then, the *logical structure induced by* $\mathfrak{M}$, denoted by $\mathcal{S}_{\mathfrak{M}} = (\mathcal{L}, \vdash_{\mathfrak{M}})$, is such that $\vdash_{\mathfrak{M}}$ is defined as follows. For all $\Gamma \cup \{\alpha\} \subseteq \mathcal{L}$,

$$\Gamma \vdash_{\mathfrak{M}} \alpha \text{ iff, for all } m \in \mathbf{M},\ \text{if } m \models \Gamma \text{ then } m \models \{\alpha\}.$$

A logical structure induced by an amst is called a *model theoretic logical structure.*

Suppose $\mathfrak{M} = (\mathbf{M}, \models, \mathcal{P}(\mathcal{L}))$ is an amst and $\mathcal{S} = (\mathcal{L}, \vdash)$ is the logical structure induced by $\mathfrak{M}$.

- If gECQ-sat holds in $\mathfrak{M}$, then gECQ holds in $\mathcal{S}$.
- If sECQ-sat holds in $\mathfrak{M}$, then sECQ holds in $\mathcal{S}$.

- If spECQ-sat holds in $\mathfrak{M}$, then spECQ holds in $\mathcal{S}$.
- If pfECQ-sat holds in $\mathfrak{M}$, then pfECQ holds in $\mathcal{S}$.

Thus, gECQ-sat, sECQ-sat, spECQ-sat, pfECQ-sat are indeed the 'semantic' analogues of gECQ, sECQ, spECQ, and pfECQ.

The interconnections between the principles discussed so far can be represented by the following diagram.

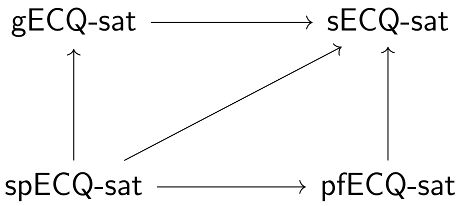

Figure 2: Semantic explosion principles—the sat variants

Since compactness of the abstract model structure is not assumed in general, we get the following variants of sECQ-sat, spECQ-sat, and pfECQ-sat by changing satisfiability to finite satisfiability. These are as follows.

Suppose $\mathfrak{M} = (\mathbf{M}, \models, \mathcal{P}(\mathcal{L}))$ is an amst.

- sECQ-finsat holds in $\mathfrak{M}$ if, for all $\alpha \in \mathcal{L}$, there exists $\Gamma \subseteq \mathcal{L}$ such that $\Gamma \cup \{\alpha\} \subsetneq \mathcal{L}$ and $\Gamma \cup \{\alpha\}$ is not finitely satisfiable.
- spECQ-finsat holds in $\mathfrak{M}$ if, for all $\Gamma \subsetneq \mathcal{L}$, there exists $\alpha \in \mathcal{L}$ such that $\Gamma \cup \{\alpha\} \subsetneq \mathcal{L}$ and $\Gamma \cup \{\alpha\}$ is not finitely satisfiable.
- pfECQ-finsat holds in $\mathfrak{M}$ if, for all $\Gamma \subsetneq \mathcal{L}$, there exists $\Delta \subsetneq \mathcal{L}$ such that $\Gamma \subseteq \Delta$ and Δ is not finitely satisfiable.

The postfix finsat in the names of the above principles indicate that these are formulated in terms of finite satisfiability.

Clearly, if the amst under consideration is compact, then the -finsat variants are equivalent to their corresponding -sat counterparts. Thus, in the presence of compactness the implications that exist between the -sat principles will hold between the corresponding -finsat principles.

The Figure 3 shows all the connections between all the seven semantic explosion principles discussed here.

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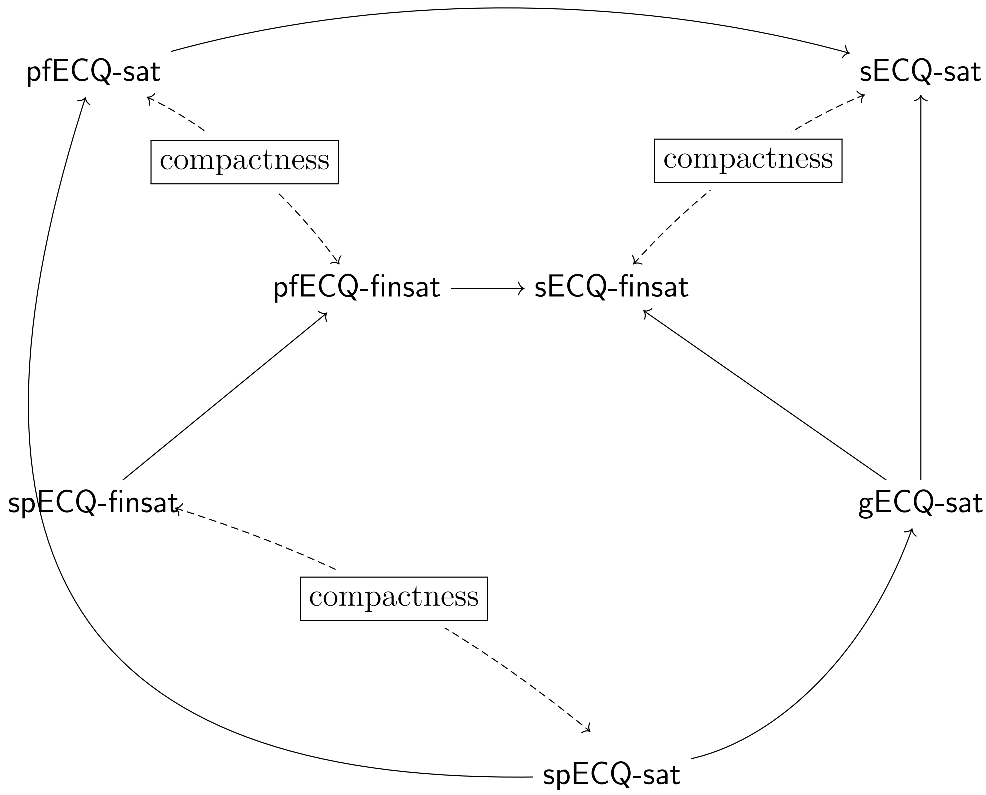

Figure 3: Semantic explosion principles

What is the Message of Meta-Theorems? A Small Case Study

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Tags: philosophical logic, philosophy of logic.

Taking the Craig Interpolation Theorem and the Beth Definability Theorem for first-order logic as running examples, we discuss a number of issues at the interface with philosophy. What do these famous theorems say about such topics as informational consequence, definability/reducibility or the role of questions? We demonstrate the interplay of logic and philosophy here, and raise a number of new conceptual and technical issues.

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The Development of Paraconsistent Epistemology and Its Capabilities

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Joint work with Juliana Bueno-Soler and Juan C. Agudelo-Agudelo

Tags: epistemic logic, paraconsistent logic, semantics of modal logic.

The logic **LFI1** (cf. [2]), designed to handle applications such as evolutionary databases, belongs to the family of Logics of Formal Inconsistency (LFIs)—systems that internalize the notions of consistency and inconsistency within the object language. This logic is equivalent to the three-valued system **J3**, introduced in [3], a natural paraconsistent logic proposed to satisfy S. Jaśkowski's criteria.

The present work extends **J3** in its **LFI1** formulation by incorporating modal operators (as in [1]), thereby laying the groundwork for a paraconsistent epistemology capable of modeling the reasoning of agents who operate coherently in the presence of contradictions. The resulting framework shows that paraconsistent agents may accept the existence of contradictions without necessarily believing them. This epistemic attitude mirrors the mindset of scientists engaged in the construction and revision of scientific theories.

By enriching the system with appropriate modalities, we distinguish three epistemic attitudes: secured knowledge, revisable knowledge, and skeptical (or open) knowledge. Secured knowledge is standard: it holds in all accessible worlds and is thus stable and consistent. Revisable knowledge is true in some but not all accessible worlds; it admits contradictions, but the agent is aware of this inconsistency, which is precisely why this knowledge is revisable. Skeptical knowledge applies when the agent holds some information but lacks any basis for assessing its consistency or inconsistency. This refinement of standard epistemic attitudes is enabled by the distinction between contradiction and inconsistency, a distinction that lies at the core of the Logics of Formal Inconsistency (LFIs).

We argue that agents endowed with this epistemic arsenal more closely approximate human reasoners and are even capable of avoiding logical omniscience through the analytic tools provided by paraconsistency.

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Metainferential Conceptions of Logical Consequence

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Tags: philosophy of logic, proof theory, substructural logic.

In this talk, I will review the motivation for endorsing a Tarskian account of logical consequence as a (structural) relation between a set of premisses and single conclusions, satisfying reflexivity, monotonicity, and transitivity.

I will argue that this conception remains defensible—perhaps indispensable [2]—even in the substructural age [3], in which many are inclined to accept sub-Tarskian of consequence relations (non-transitive, non-monotonic, or multi-conclusion). The turn away from the Tarskian model is premature. While substructural logics rightly challenge some assumptions embedded in classical reasoning, they do not necessitate abandoning the Tarskian conception of consequence. Instead, they invite a reinterpretation of the domain over which Tarskian properties apply and a metainferential conception of logic has the resources to provide this reinterpretation (see, e.g., [1, 2]).

This reconceptualization preserves the Tarskian properties of consequence while accommodating substructuralism. I will argue that the Tarskian conception, once recontextualized metainferentially, is not only compatible with embracing substructural logics but provides a rich framework for understanding substructurality.

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Substitutions of Variables are Finitely Axiomatizable Over Quantifications and Permutations

Zalán Gyenis

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Tags: algebraic logic.

In this talk I survey some of the previous results around (non-)finite axiomatizability of logical connectives, and present a new result, joint with Hajnal Andreka and Istvan Nemeti, that states that the equational theory of the class of finite dimensional representable polyadic algebras is finitely axiomatizable over its substitution-free reduct. In the logical parlance, substitutions of variables in finite variable first-order logic can be described by finitely many axioms over the Boolean operations, existential quantifiers and permutations of variables.

Ontology of Leśniewski in the Proof-Theoretic Setting

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Tags: history of logic, history of Polish logic, non-classical logics, philosophical logic, proof theory.

Stanisław Leśniewski is one of the most outstanding Polish logicians. His systems (protothetics, ontology, mereology) belong to the most original and important results of Lvov-Warsaw School. In the talk I would like to focus on Leśniewski's ontology (LO), the expressive calculus of names. It provides a basis for mereology and enables more direct formalisation of reasoning in natural languages. Interestingly, it appeared that LO may be fruitfully investigated by means of proof-theoretic methods. In particular, its elementary part was characterised by means of the analytic sequent calculus satisfying interpolation. Moreover, the application of proof-theoretic techniques allowed me to obtain its extended version ELO which introduces lambda terms to represent complex descriptive names. In the talk I will present a hierarchy of sequent calculi adequate w.r.t. several variants of LO and ELO and discuss their important properties like cut elimination, the subformula property and interpolation.

A Multi-Agent Logic for Reasoning About Duties and Powers in Private Law

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Joint work with Tianwen Xu

Tags: deontic logic and action.

Duties and powers are two fundamental notions in private law. In this work, we provide our conception of duties and powers and present a logic for reasoning about them. We treat duties as agents' obligations towards others to perform actions. We think that powers are agents' legal abilities, conferred by law, to change legal positions between agents. Powers are exercised by manifestation of intension. The logic has an ontic level and a normative level. The ontic level of the logic is a multi-agent dynamic logic, where agents have abilities to change atomic facts. At its normative level, agents have duties towards others to change atomic facts, and have powers to change duties by speech acts and changing atomic facts. We study the implications of the logic, compare it to some related work, and show its completeness.

Obligations for Sequentially Composed Actions

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Tags: deontic logic, logic of action, philosophical logic.

This work provides a formal presentation of sequentially composed actions and their normative aspects. It is grounded in an algebraic framework that accounts for both successful and failed enactments of actions, as presented in [3]. On this basis, we delineate two separate frameworks of obligation concerning sequentially composed actions. One of them adheres to local norms that govern one-step actions, reflecting the model discussed in [3]. The other one evaluates the results of entire action sequences in the context of an agent's objectives, paralleling the methodology in [1] and inspired by Andersonian-Kangerian reduction strategies. This exploration broadens previous research on the deontic properties of actions and states, as explored in [4, 5], specifically focusing on sequentially composed actions. Local norms underpin action-based obligations, while goal-oriented norms align with state-based obligations.

For norms based on local contexts, we apply the Standard Deontic Logic model for its clarity and brevity, as noted in [6]. Obligation in these settings is based on prohibition: one simply ought to refrain from actions that are prohibited.

Regarding norms oriented towards goals, the primary concept is that in every situation an agent faces, there are optimal states the agent ought to aim for. Norms for sequentially composed actions are then derived based on whether these optimal states are achieved by a sequence of actions or not. We examine several ways of establishing obligations, as preliminarily sketched in [2].

The formalism is evaluated against the ways in which sequentially composed actions are employed in practical reasoning. We define contexts in which sequences of actions serve to describe various activities and to capture possible meanings of norms within those contexts. Finally, we discuss whether our formalism is suitable for this type of reasoning.

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Partial Algebras and Logic

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Joint work with Janusz Czelakowski

Tags: algebraic logic.

This presentation introduces a new perspective on logical consequence over partial algebras [1]. We discuss nontrivial consequence-like relations that challenge the classical Tarski–Łoś–Suszko paradigm. Our approach is particularly motivated by the behavior of partial Boolean algebras in the Kochen–Specker approach to quantum theory ([3] and [4]). Yet another motivation stems from linguistics and the need of formulation of a uniform theory of semantic fields. We employ tools from Abstract Algebraic Logic to define and investigate different notions of consequence relations with respect to the classical properties of Tarskian consequence relations. Although these systems are not necessarily Tarskian consequence operations, they retain some interesting meta-logical properties.

We study in more depth one of those systems, providing a system of inference rules that syntactically characterizes the logical behavior observed in quantum contexts, such as the breakdown of commensurability and the failure of total truth assignments [2].

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How Can Fixed Points Express Change?

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Tags: logic and computer science, fixed-point calculus, model theory.

Fixed-point definitions naturally generalise induction and co-induction. They help us to understand the interplay between static and dynamic in various processes studied in computer science. In particular, they capture infinite behaviour of finite-state automata, which was a mathematical tool used by Buchi and Rabin in the 1960s to show decidability of monadic second-order arithmetic of n successors. Fixed-point definitions usually rely on the Knaster-Tarski theorem on complete lattices. However, a fixed-point paradigm can go beyond that. In a recent work with Paweł Parys and Michał Skrzypczak, we have shown a probabilistic version of the Rabin Tree Theorem: the question if a random tree satisfies a sentence of Rabin's arithmetic is also decidable. To this end, we view the states of an automaton as random variables and develop a fixed-point calculus over the structure of their joint distributions.

Formality Without Invariance

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Tags: philosophical logic, philosophy of logic.

Given the significant influence of Tarski's work on the demarcation of logical operations (cf. [1, 2]), it is widely believed that the defining characteristic of logicality is *formality*—the inability to discriminate among individuals or otherwise be sensitive to their distinguishing features—and that formality should be understood in terms of invariance under arbitrary substitutions of individuals. But all extant invariance accounts face well-known problems. There are strong reasons to think problems for these accounts arise from their focus on semantic values. I propose instead a purely proof-theoretic account of formality—one that dispenses with any appeal to semantic values or to invariance—and thereby avoids the difficulties that beset its predecessors.

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On Universally Free Versions of Nelson's N4 and the Logic of First Degree Entailment

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Joint work with Henrique Antunes

Tags: information-based logic, non-classical negation, paraconsistent logic.

In this paper we present universally free versions of the logic of first-degree entailment (*FDE*) and Nelson's logic *N4*, called *FFDE* and *FN4*, respectively. *FDE* is the implication-free fragment of the Anderson and Belnap's system E of entailment, but has been studied since the 1970s by Belnap and Dunn as an information-based logic. *N4* is known as the paraconsistent Nelson logic, and has been widely studied as a logic suitable for contexts of empirical investigation and information processing. *FFDE* and *FN4* are equipped with an identity predicate capable of representing inconsistent and incomplete scenarios, with independent positive and negative rules, analogous to the rules of *FDE* and *N4*. Identity is interpreted as a congruence relation, which is weaker than classical identity, but suitable for the information-based interpretation. We argue that, as far as Tarskian logics are concerned, this reading of identity, together with empty domains, languages with uninterpreted constants, and variable domains, fits the idea of an information-based logic proposed by Belnap and Dunn better than the non-free versions studied so far.

Polish Roots of Meredith's System of Modal Logic

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Tags: history of logic, history of Polish logic, modal logic.

Carew Arthur Meredith, an Irish logician, was influenced by Jan Łukasiewicz, whom he met in Dublin, and Łukasiewicz's student Mordechaj Wajsberg. My talk will highlight how Łukasiewicz's four-valued modal logic and Wajsberg's extended class calculus affected Meredith's own modal system and his property calculus, later known as U-calculus. The better understanding of the connection between the systems of these three logicians could shed light on some troublesome features in these systems.

Łukasiewicz [6, pp. 391–392], a pioneer of many-valued logic, introduced a four-valued system in 1953. Unlike standard modal systems, his logic was extensional and did not rely on possible worlds semantics [6, pp. 370–371]. The system used two modal operators for possibility (Δ and ∇) and one for necessity ($\Box$), with the notable feature that no apodictic statement could be true in the system [5, p. 169]. This approach aimed to avoid modal paradoxes, such as those raised by Quine [13, p. 47] and [10, §160] Łukasiewicz's rejection of the necessitation rule (if α is provable in the system (i.e., is a theorem for the system), then $\Box\alpha$ is provable in the system (i.e., is a theorem of the system)) and his insistence on extensionality [7, p. 261] made his system controversial (see e.g., [11, p. 3], [14]). Nevertheless, it laid the groundwork for Meredith's later developments.

The second part of my talk will concern, Wajsberg's property calculus. Mordchaj Wajsberg developed a class calculus equivalent to Lewis's system of strict implication and provided its first semantic completeness proof [16, p. 52]. Historians of the Lvov-Warsaw School consider this Wajsberg's result as the first proof of semantic completeness of Lewis' calculus of strict implication (see [17, p. 134], [15, p. 9]). His system interpreted modal operators through quantification, influencing Meredith's property calculus [2, p. 379].

Thirdly, I will present Meredith's system of modal logic. Meredith introduced his modal system in 1953, though it was published only in 1965 [9]. His system is four-valued, using constants 'n' (contingently true) and 'ṅ' (contingently false), and includes a functional variable δ. Like Łukasiewicz's, it is extensional and avoids the necessitation rule for formulas involving 'n' [8, p. 118]. The system's semantics are matrix-based, and the truth-value 'n' is interpreted as "true in this world, false in others," aligning with Prior's notion of "the totality of what is the case" [9, pp. 99–101].

In contrast to his system of modal logic, Meredith's property calculus was unpublished during his lifetime. It was later rediscovered by [3]. It interprets propositional variables as predicates and introduces a binary relation 'U' [12, p. 133]. This system served as a semantics for modal logics **T**, **S4**, and **S5**, and influenced Prior's development of hybrid logic [4]. The contingent constant 'n' is defined in property calculus as a property of a specific object 'a', linking it to the semantics of possible worlds without invoking intensional entities [8, pp. 120–121].

To conclude, Meredith's modal logic and property calculus were deeply shaped by Łukasiewicz's extensional modal logic and Wajsberg's property calculus. While Meredith's work remained underappreciated for decades, recent scholarship (e.g., [1], [4]) has highlighted its significance, especially in the context of hybrid logic. This renewed interest may also lead to a deeper appreciation of Wajsberg's contributions, whose legacy deserves further exploration.

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Conditional Expressivity and Collective Deontic Admissibility

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Joint work with Frederik Van De Putte and Hein Duijf

Tags: deontic logic and action, modal logic, model theory.

We introduce and study a novel variant of the standard concept of expressivity. Standardly, a statement ϕ from a given language is *expressible* in a sublanguage of that language if and only if there is a statement ψ in the sublanguage such that ϕ and ψ are logically equivalent. We say that a statement ϕ from a given language is *conditionally expressible* in a sublanguage of that language if and only if there are subsets Γ and Δ of the sublanguage such that Γ non-trivially implies that ϕ and Δ are logically equivalent. In the first part of this talk, we take the perspective of universal logic and characterize our novel concept of conditional expressivity both syntactically and semantically. In particular, we show that if ϕ non-trivially implies or is non-trivially implied by any of the statements of the sublanguage, then ϕ is conditionally expressible in that sublanguage. (A statement ϕ *trivially implies* a statement ψ if ψ is a logical truth. A statement ϕ *is trivially implied by* a statement ψ if ψ is a logical falsity.) Consequently, if ϕ is not conditionally expressible in the sublanguage, then there is no statement in that sublanguage that non-trivially implies or is non-trivially implied by ϕ.

Although the universal logic perspective shows it to be widely applicable, our concept of conditional expressivity was originally motivated by the philosophical debate on collective agency, collective obligations, and collective responsibility.[1] That debate has been focused almost exclusively on the question whether and how statements about groups are inferentially related to statements about individuals. It has thereby prevented another question from being asked: whether and how statements about a given group are inferentially related to statements about *other groups*. In the second part of this talk, we rephrase the latter question using our concept of conditional expressivity, focusing on *collective deontic admissibility statements* of the form "Group $\mathcal{G}$ of agents performs a deontically admissible group action" (formalized as $\star_{\mathcal{G}}$). Such statements are a key component of a well-established *deontic logic of collective agency* that models actions, omissions, abilities, and obligations of finitely many individuals and groups of individuals.[2]

In the present study, we assess the inferential relations between, on the one hand, the collective deontic admissibility statement $\star_{\mathcal{G}}$ about a group $\mathcal{G}$ of agents and, on the other, collective deontic admissibility statements about $\mathcal{G}$'s subgroups, supergroups, outgroups

[1] Key contributions to the debate include [10], [8], [11], [15], [3], and [12].

[2] The deontic logic of collective agency [14] is a deontic logic in the tradition of *stit* ('sees to it that') logics of agency. See [1] and [7] for textbook presentations of *stit* logics and historical references. Collective actions and/or obligations have been studied using *stit*-like frameworks by [6], [16], [1, Ch. 10], [7, Ch. 6], [9], [2], [5], [13], and [4].

and partially overlapping groups. In particular, we define a natural class of sublanguages of the full language of the deontic logic of collective agency and assess, for each sublanguage in that class, whether $\star_{\mathcal{G}}$ is conditionally expressible in that sublanguage, that is, whether $\star_{\mathcal{G}}$ non-trivially implies or is non-trivially implied by any of the statements in that sublanguage.

Our central theorem on collective deontic admissibility establishes that the collective deontic admissibility statement $\star_{\mathcal{G}}$ is conditionally expressible in a given sublanguage from the natural class of sublanguages of the full language of the deontic logic of collective agency *if and only if* that sublanguage contains a collective deontic admissibility statement $\star_{\mathcal{H}}$ for some supergroup $\mathcal{H}$ of $\mathcal{G}$. The right-to-left direction of our central theorem is new. Its left-to-right direction is a considerable strengthening, in various ways, of an earlier result from [5]. Phrased differently, it says that if the sublanguage does not contain a collective deontic admissibility statement $\star_{\mathcal{H}}$ for some supergroup $\mathcal{H}$ of $\mathcal{G}$, then there are no non-trivial inferential relations between $\star_{\mathcal{G}}$ and any statement of that sublanguage.

Our paper proceeds as follows. We first take the perspective of universal logic to define conditional expressivity and establish necessary and sufficient conditions for it. We then study the concept of conditional expressivity semantically and give a semantic criterion for proving that a statement is not conditionally expressible in a given formal language. Next, we present the full formal language and the semantics of the deontic logic of collective agency. We define a natural class of sublanguages that differ with respect to (a) the included deontic admissibility statements and (b) the included *stit* operators for individual and group agency. We then give conditions under which two models are bisimilar with respect to one of the sublanguages in the class and prove a general Hennessy-Milner theorem that covers every sublanguage in the class. Building on the previous steps, we generalize the impossibility result on conditional expressivity from [5] to much more expressive sublanguages. We prove that the collective deontic admissibility statement $\star_{\mathcal{G}}$ is conditionally expressible in a sublanguage from the natural class of sublanguages if and only if that sublanguage contains a collective deontic admissibility statement $\star_{\mathcal{H}}$ for some supergroup $\mathcal{H}$ of $\mathcal{G}$. Lastly, we compare our new impossibility result with the earlier result from [5] and discuss two straightforward applications.

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Input/Output Logic for Norms and Trust-Aware Information Filtering

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Tags: deontic logic and action, logic and artificial intelligence, modal logic, non-classical logic, philosophical logic.

Imagine a device that accepts propositions as input and emits propositions as output. Classical consequence fits this picture, but only as a special case: every input is also an output (by reflexivity), and inference is, in a sense, reversible (by contraposition). Many interesting devices lack these features, and they fall, roughly, into two broad families.

In the first family, inputs are conditions and outputs state what is desirable under those conditions. The desiderata may be obligations, ideals, goals, intentions, or preferences. A condition we consider may itself be far from desirable, so inputs need not be outputs; moreover, contraposition is inappropriate for conditional goals.

Over the last quarter-century, input/output (I/O) logic has turned this perspective into a toolkit: core out-operations (out1-out4); principled handling of contrary-to-duty reasoning via constraint selection and, more recently, built-in consistency checks; a refined theory of permission (negative and several positive kinds); variants avoiding output-weakening; proof systems (sequent calculi), complexity baselines, and networked architectures (logical input/output nets) that separate constitutive from regulative components and support automation and applications.

The second family filters information: the box may block some inputs and let others through, possibly in a qualified form. Inputs can be source-tagged reports ("according to source i, x"), and outputs may be x itself, a hedged version of x, or nothing—depending on who i is. One may require quorums ("at least two independent sources"), or transform market data into an analyst's commentary, or date-and-place of birth into astrological predictions. Here outputs represent belief or expectation rather than obligation.

This talk surveys the state of the art and then discusses ongoing research with Xu Li and Liuwen Yu to develop a trust-aware I/O account of filtering.

From “We” to Logic: Pathways Into Collective Agency Dependence, Norms, and Coalitional Power

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Tags: philosophical logic.

This talk examines how to formally capture the notion of collective agency. Traditional debates have oscillated between reducing collectives to individuals and recognizing them as genuine agents. We propose a relational perspective: collective agency arises from stable patterns of dependence among members, rather than from the individuals themselves [1].

Three formal pathways are explored. First, we extend the logic of functional dependence to model interdependencies in choices, highlighting stability conditions such as Nash equilibrium and Pareto optimality [2]. Second, a utility- and norm-based approach shows how social norms reshape strategic interaction and sustain collective outcomes [3]. Third, an effectivity and coalitional power perspective revises coalition logic to account for endogenous stability, exogenous powers, and hierarchical structures [4].

These approaches reveal collective agency as a dynamic phenomenon grounded in relational structures, normative commitments, and coalitional capabilities, thereby connecting logic, game theory, and social ontology within a unified framework.

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Reasoning About Questions and Reasoning with Questions

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Tags: logic of questions, non-classical logic.

One can reason *about* questions, as in:

> *The question "How did you solve this problem?" has no answer since you did not solve it. Therefore the question "When did you solve the problem?" is pointless.*

but one can also reason *with* questions, for instance:

> *Your solution to the problem is breathtaking. When did you solve the problem?*

Inferential Erotetic Logic (IEL for short) provides a formal account of erotetic inferences, that is, inferences in which questions play the role of conclusions and possibly premises. In particular, IEL explicates the intuitive notions of validity of erotetic inferences and characterizes semantic relations which ensure validity of such inferences. Although IEL pays most of its attention to reasoning with questions, some concepts provided by IEL are useful in analysing reasoning about questions as well.

In my talk I will present the basics of IEL and I will show how IEL copes with reasoning with questions and how the conceptual apparatus of IEL can be used in the case of reasoning about questions. I will also point out certain open problems of IEL.

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General Session

Contributed Lectures

Protagoras' Logic in Plato's *Protagoras*

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Tags: ancient logic, history of logic, Plato, Protagoras, Sophists.

Plato's dialogue *Protagoras*, alongside the *Theaetetus*, is one of the most important sources of knowledge regarding the philosophical views of the sophist Protagoras. Despite Plato's aversion to the sophistic movement, Protagoras is presented in a positive light, as one of its most prominent representatives. The confrontation between the sophist and Socrates in the dialogue appears highly charged, and its outcome remains ambiguous.

Both the content and the composition of the dialogue are the subject of numerous analyses. They examine Socratic philosophical tropes, the distinction between the Platonic and the Socratic, and finally, Protagoras's philosophical, social/political, and pedagogical beliefs. Researchers have drawn attention to the fact that Protagoras's quoted statements differ in style from the rest of the dialogue. Experts believe that the Platonic dialogue contains either authentic fragments (e.g., Ulrich von Wilamovitz-Moellendorf) or conscientious paraphrases of Protagoras's own words (e.g., Mario Vegetti, Mauro Bonazzi).

The extensive literature on the subject analyzes, on the one hand, the Protagorean concept of "dexterity" (euboulia), fluency in speech, and discusses the teaching of virtue. On the other hand, dialectical tropes and the layers of argument presented in the dialogue are examined. The key fragment of the Platonic dialogue (due to the character of Protagoras) is the so-called Great Speech of Protagoras (Prot. 320c-328d). This speech consists of two parts: the myth of Prometheus (a parable) and a rhetorical part, based on logos (supplementary argumentation). Interestingly, scholars have focused on the first part, usually treating the second part superficially or as a supplement/support to the first.

My attention focuses on the second part of Protagoras's speech, based on logos. The starting point for my analysis is the assumption that Protagoras not only claims that speaking skills, making a weaker argument stronger, can be taught, but that this skill is actually used by him. I assume that since the dialogue discusses the presentation of proof (320c, 323a), since Protagoras says he will provide proof (324d), and since the proof (proof/strict argument) has been provided (328c), it is worth seeking this proof. Of particular note is the fragment where Protagoras states that the conclusion/inference is necessarily imposed (325d). In my work, I analyze Protagoras's forms of speech from a strictly formal perspective, abstracting from their content. I will propose several paraphrases of Protagoras's reasoning, arguing that we are dealing here with the necessity of accepting the conclusion/inference/thesis being proven, which is a consequence of using a correct/reliable inference scheme. Thus, I see this study as supporting W. and M. Kneale's thesis that the ability to recognize and accurately use formally correct inferences preceded the work of Aristotle [4]. To fully capture the essence of Protagoras's teaching, Protagoras's great speech will be compared to the Socratic dialogue (Prot. 328d-335c).

The analysis provides further interesting findings regarding the formal tools used by Protagoras. It also sheds new light on the Sophistic workshop in general and how it was perceived by Plato.

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Philosophical and Formal Abduction: A Comparative Study

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Tags: abduction, formal theories of truth, general methodology of science, philosophy of science.

Abduction is a mode of inference that is pivotal in philosophy. More specifically, realists argue that both scientific inquiry and everyday problem-solving are conducted through the use of multiple successive instances of abductive syllogisms. On a second level, the realist program itself is defended via the No Miracle Argument, which constitutes a higher-order abduction. Contemporary consensus holds that what is commonly referred to as "philosophical abduction" encompasses two complementary methods: the heuristic and the selective. The latter is typically identified with Inference to the Best Explanation (IBE).

In recent decades, there has been an effort to formalize IBE. Notably, in the 1980s, researchers attempted to frame IBE as a specific instance of the set covering problem. Over the following decades, work by Aliseda, Kowalski, Kakas, Meheus, Magnani, and others led to the development of the AKM framework. This framework defines the abductive problem and outlines some of the conditions that should be met for an algorithm to be considered as having successfully solved it.

More recently, numerous formalizations based on the AKM framework have been proposed. These use a wide variety of logical languages—including classical propositional logic, non-classical propositional logic, first-order logic, and first-order modal logic—in order to find either minimal or explanatory solutions.

Today, IBE is generally understood to comprise two distinct branches: philosophical IBE and formal IBE. In this talk, their commonalities as well as their differences will be explored. This will be done using the concepts of strong equivalence and weak equivalence relation. Two methods are said to be strongly equivalent if every element that plays a substantive role in the functioning of one method has a corresponding counterpart in the other. Conversely, they are weakly equivalent if, given the same input problem, they consistently produce the same output solutions.

Regarding strong equivalence, it will be shown that although philosophical and formal IBE share certain structural features, they are not fully equivalent. In particular, there are explanatory virtues that are considered critical in philosophical IBE but have not yet been formalized in models based on the AKM framework. Similarly, it will be argued that even if philosophical IBE is treated as a black box, it is not weakly equivalent to its formal counterpart, since there are several inputs for which the two methods produce divergent outputs.

Furthermore, it cannot be claimed that one method is strictly more general than the other—for example, that formal IBE fully resolves a range of problems addressed by philosophical IBE. While the two methods may successfully solve a subset of genuine abductive problems in similar ways, at the same time, their methodologies and the solutions they produce for other problems radically differ.

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Reflexions on Some Philosophical Aspects of Adaptive Logics

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Tags: non-classical logic, paraconsistent logic, philosophical logic, adaptive logics.

Where $\mathcal{W}$ is the set of closed formulas of a language or language schema $\mathcal{L}$ and $\wp(\mathcal{W})$ is the power set of $\mathcal{W}$, a logic[3] $\mathbf{L}$, defined over $\mathcal{L}$ is a function $\mathbf{L}\colon \wp(\mathcal{W}) \to \wp(\mathcal{W})$ that maps every premise set $\Gamma \subseteq \mathcal{W}$, to a unique *consequence set*, $Cn_{\mathbf{L}}(\Gamma) \subseteq \mathcal{W}$.[4] A logic $\mathbf{L}$ is *deductive* iff it is reflexive, transitive and monotonic. A deductive logic $\mathbf{L}$ defined over the language $\mathcal{L}$, may be seen as defining the logical symbols of $\mathcal{L}$ and $Cn_{\mathbf{L}}(\Gamma)$ as comprising the members of $\mathcal{W}$ that follow from Γ in view of the meaning of the logical symbols of $\mathcal{L}$. Examples of deductive logics are familiar enough. Some non-deductive logics $\mathbf{L}$ are called *defeasible* because reasonings (or derivations) that proceed in terms of such $\mathbf{L}$ have the following property: a formula displayed at a certain line l of the derivation may at a later point[5] in the derivation be considered as non-derivable at a later stage in view of formulas displayed at lines of that stage.[6]

By a *method* I shall understand a set of operations that bring us from a situation s_1 to a situation s_2. If $s_1 \cup s_2 \subseteq \mathcal{W}$, then there obviously is a logic $\mathbf{L}$ such that, $s_2 \subseteq Cn_{\mathbf{L}}(s_1)$; otherwise, it is clearly possible to choose a language $\mathcal{L}$ (with set of closed formulas $\mathcal{W}$) and a logic $\mathbf{L}$ in such a way that $\Gamma_1 \subseteq \mathcal{W}$ adequately describes s_1 and $\Gamma_2 \subseteq \mathcal{W}$ adequately describes s_2 and $\Gamma_2 \subseteq Cn_{\mathbf{L}}(\Gamma_1)$ just in case applying the method to s_1 brings one to s_2. In this case $\mathbf{L}$ is said to *characterize* $\mathcal{W}$. As examples of methods, I mention: interpreting theories as consistently as possible [1, 3], inductive generalization [2] and abduction [4].

In view of the above definitions, every method is characterized by a logic. To describe the matter in a simple way, the conceptual framework in which the domain of a discipline is expressed by a given theory may be identified with a certain language. The empirical data are described in terms of that language and the (empirical) theorems of a specific theory $\mathcal{T}$ organizing the domain are a subset of the corresponding $\mathcal{W}$—usually a theory will be given as a couple $\langle \mathcal{A}, \mathbf{L}\rangle$, where $\mathcal{A}$ is the set of non-logical axioms, $\mathbf{L}$ is a deductive logic and the theorems of $\mathcal{T}$ are identified with $Cn_{\mathbf{L}}(\mathcal{A})$. The distinguishing feature of

[3]"Logic" is here defined in the broadest sense of the term.

[4]As is usual, $Cn_{\mathbf{L}}(\Gamma)$ abbreviates $\{A \in \mathcal{W} \mid \Gamma \vdash_{\mathbf{L}} A\}$.

[5]Describing this in a technically precise way requires the introduction of stages (sequences of lines) of proofs, a proof being a chain of stages.

[6]This relates to the essential property of defeasible logics, viz. that there is no positive test for derivability by such logics, which means that the set of derivable formulas is not recursive—the relative complement $\mathcal{W} \setminus Cn_{\mathbf{L}}(\Gamma)$ may however be recursive. For adaptive logics *in standard format* the property of derivations is made technically precise in terms of *abnormalities*—see below in the text—and *marks* governed by a *marking definition*, which depends on $\mathbf{L}$'s *strategy*. The formula displayed on a marked line is considered as not-derived at that stage of the proof. Incidentally *LPm* [5] is not an adaptive logic in standard format; also, it has no derivations: no derivations were ever defined and all attempts I made to define them turned out abortive.

a defeasible logic **L** is that, as the reasoning in terms of the logic (or, alternatively, as explicated by **L**) goes on, a conclusion drawn at some point of the reasoning may be revoked.

Any formula of the native language of a logic may function as an abnormality. If **L** is deductive, its consequence sets that contain an abnormality are trivial. If **L** is defeasible, it interprets the premise set in such a way that (in some or other sense) 'minimizes' abnormalities (in function of the adaptive strategy).

Within the preceding framework, I shall argue that any (deductive or defeasible) logic has a multiplicity of variants possibly justified by widely diverging considerations, and that it is essential to choose the variant that is suitable for the context in which one wants to apply it.

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Karl Schröter (1905–1977) and His Interest in Polish Mathematical Logic: Research Based on Unpublished Archival Documents

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Karl Schröter worked in the field of mathematical logic and foundations of mathematics, first at Westphalian Wilhelm University of Münster (*Westfälische Wilhelms-Universität Münster*), then at the Berlin Humboldt University (*Humboldt-Universität zu Berlin*). From today's perspective, he is considered the most important mathematical logician in the German Democratic Republic [1]. He founded the Logic School in Berlin and developed mathematical logic in that country. Most academics working in East Germany in mathematical logic and foundations of mathematics were either direct students of Schröter or studied under his intellectual influence. He referred to his Münster's supervisor, Heinrich Scholz (1884-1956), and representatives of the Polish School of Mathematical Logic as his teachers of mathematical logic [2].

In this lecture, we present the results of our research on two topics. First, we give—compared to what has been published so far—an extended scientific biography of Schröter and a complete list of his scientific works. Our second aim is to inform the logical community about unknown archival documents confirming Schröter's interest in the Polish logic (Jan Łukasiewicz, Stanisław Leśniewski, Alfred Tarski and Mordchaj Wajsberg). To accomplish these two tasks, we conducted archival research in the Münster and Berlin archives, performed a preliminary analysis of the collected documents, and compared our findings with existing publications on Schröter. The archival research made it possible to clarify his scientific curriculum vitae. The documents also indicate areas for further, more detailed research, including those concerning Schröter's connections with the Polish School of Mathematical Logic.

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In Defense of Quantifier Generalism: Holism and Infinitary Resources

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Tags: model theory, philosophical logic, philosophy of science.

According to quantifier generalism, all facts about the world can be expressed in a language devoid of proper names, whose only referential expressions are variables bound by quantifiers. In this talk I will consider and repel some of the recently raised objections against this position. The central part of the talk is a critical analysis of the claim, advanced by Ted Sider, that quantifier generalism is inevitably holistic, and therefore requires unusually strong expressive resources when applied to infinite domains. Using an example of arithmetic, it will be shown that there is a simple generalistic description of natural numbers that does not resort to any infinitary conjunctions or quantifiers. Such generalistic accounts also exist in many cases involving continua (such as descriptions of matter distribution in a continuous space). Moreover, these accounts are arguably superior to their individualistic counterparts due to their parsimony. In addition to that, Sider's argument alleging that generalism cannot account for the difference between non-isomorphic models of arithmetic is repelled.

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Symmetry-Breaking Indeterminism as a Challenge to Generalism

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Tags: philosophical logic, philosophy of science, semantics of modal logic.

The debate between individualism and generalism (qualitativism) concerns, inter alia, the question of whether qualitative identity implies numerical identity. One important argument against the affirmative answer to this question is based on an analysis of cases known as symmetry-breaking indeterministic processes, including the case of a collapsing tower. Generalism implies that all processes of that kind are deterministic, contrary to our snap judgment. This talk will provide a multifaceted analysis of the above argument against generalism, taking into account its metaphysical, scientific and modal aspects. It is suggested that the intuition of indeterminism regarding the considered cases can be explained by a distinct phenomenon of indeterminacy affecting the diachronic identity of participating objects. It is also observed that both individualism and generalism are unable to formulate a theory which could make definite predictions that are more specific than the ones made in purely qualitative language. This strongly suggests that the purported indeterminism of the considered scenarios is not genuine. Finally, the modal analysis shows how generalism is capable of expressing the idea of alternative possibilities without violating the fundamental assumption of determinism. This approach is based on the Lewisian counterpart semantics for quantified modal logic with same-world counterparts.

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Kotarbińska, Lutmanowa, Romahnowa, Rasiowa. Women from the Lvov-Warsaw School in Polish Logic

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Tags: history of logic, history of Polish logic.

Shortly after World War II, three Polish chairs of logic were held by women: in Łódź (and later in Warsaw) by Janina Kotarbińska, in Wrocław by Maria Kokoszyńska-Lutmanowa, and in Poznań by Seweryna Łuszczewska-Romahnowa. Soon afterward, another second chair of logic in Warsaw was taken by a fourth woman: Helena Rasiowa. Such a significant participation of Polish women in research and teaching in logic is a phenomenon on a world scale.

The scholars mentioned had diverse research interests, ranging from mathematical logic to the methodology of science and logical semiotics. What united them was their intellectual lineage from the Lvov–Warsaw School, the most outstanding Polish philosophical-logical school. Kokoszyńska and Romahnowa were students of Kazimierz Twardowski in Lwów. Kotarbińska and Rasiowa were representatives of the Warsaw branch of the school.

In the paper, I present the scholarly and teaching achievements of Polish women logicians and briefly explain the sociological phenomenon of such a strong female representation in Polish logic.

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Some Remarks on Univocal Connectives

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The paper [2] deals with the univocity of intuitionistic and classical connectives in the context of usual axiomatic settings for those logics. Some models were given in the case of non-univocity. The purpose of this talk is to provide certain algebraic considerations in order that the models are easy to understand. We also take the opportunity to simplify a derivation proving that the conditional is univocal in the classical fragment with signature $(\to, \vee)$, where $\to$ satisfies Peirce's law.

In order to prove that the (usual) conditional is not univocal in the Hilbert calculus it is enough to consider the only two up to isomorphism three-element Hilbert algebras. However, in the proof of Theorem 1 in [2] the first table defines an operation which is not a Hilbert algebra operation. It is an operation that results from considering the class of algebras $(A; \to, 1)$ with type $(2, 0)$ satisfying the following conditions:

$y \to (x \to y) = 1$,
$[z \to (x \to y)] \to [(z \to x) \to (z \to y)] = 1$,
if $x = 1$ and $x \to y = 1$, then $y = 1$ (or if $1 \to y = 1$, then $y = 1$).

In fact, the usual algebraic counterpart of the Hilbert calculus is not the class we have just defined, it is the class of the already mentioned Hilbert algebras, which may be defined adding the following condition to the conditions given above:

(O) if $x \to y = 1$ and $y \to x = 1$, then $x = y$.

In order to prove soundness and completeness it is enough to consider the (smaller) class of Hilbert algebras. The fact that a broader class of algebras, not necessarily satisfying (O), is available, turns out to be useful in order to find appropriate models to refute univocity. In the Hilbert case, we call them *pre-Hilbert algebras*. Analogously, *pre-Tarskian*, *pre-Heyting*, or *pre-Boolean* algebras may be obtained. For instance, the first table in the proof of Theorem 2 in [2] defines a conditional in a pre-Heyting algebra and the first tables in Theorems 6 and 10 in [2] define operations in pre-Boolean algebras.

Pre-Hilbert algebras appear explicitly in [3, p. 392] but called "quasi I algebras". They also appear in [1, p. 4] as a particular case of what the author calls a "pre-algebra de Hilbert".

Some of the resulting algebras may be somewhat surprising. For instance, note that the footnote to the proof of Theorem 7 in our paper mentions the existence of a six-element counter-model (it is well known that there is no six-element Boolean algebra).

Another unexpected fact is the non-univocity of the conditional in classical logic given with the language $(\to, \neg)$, where all connectives are definable. Surprisingly, that fact can be proved using two isomorphic models, those given by the tables appearing in the proof of Theorem 7 in [2].

Theorem 8 of the paper [2] states that the conditional is univocal in the $\{\to, \vee\}$-fragment of classical logic. Note that in the classical $\{\to\}$-fragment the disjunction is definable as $(\alpha \to \beta) \to \beta$. So, when adding an additional conditional $\Rightarrow$ to the $\{\to, \vee\}$-fragment, in fact three disjunctions are present, that is, the formulas $\alpha \vee \beta$, $(\alpha \to \beta) \to \beta$, and $(\alpha \Rightarrow \beta) \Rightarrow \beta$, which are interderivable. The derivation appearing in [2], which is unnecessarily complicated and, consequently, very difficult to follow, was a simplification of a derivation given by a program. We now provide a derivation which is very easy to understand.

1. $\alpha \to \beta$	Hyp
2. $\alpha \vee (\alpha \Rightarrow \beta)$	By Peirce's Law for $\Rightarrow$
3. $(\alpha \Rightarrow \beta) \to (\alpha \Rightarrow \beta)$	Derivable formula
4. $\beta \vee (\alpha \Rightarrow \beta)$	By ($\vee$E) from steps 1, 2, and 3
5. $\beta \Rightarrow (\alpha \Rightarrow \beta)$	Axiom for $\Rightarrow$
6. $(\alpha \Rightarrow \beta) \Rightarrow (\alpha \Rightarrow \beta)$	Derivable formula
7. $\alpha \Rightarrow \beta$	By ($\vee$E) from steps 4, 5, and 6.

Note the mixed formula at step 3.

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Da Costa's C-Systems: A Look Beyond the Surface

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Tag: paraconsistent logic.

In my presentation, I will discuss several strategies for extending da Costa's calculus C_1 in a way that avoids collapsing into classical propositional logic. My primary focus will be on the gently paraconsistent extensions, namely those that admit the principle of gentle explosion. The subsequent discussion will examine da Costa's calculus in the context of De Morgan's laws and intuitionistic implication. I will propose a weakening of C_1 such that its negation-free fragment corresponds to positive intuitionistic propositional logic. The weakening exhibits several significant features, one of which is that extending it with the law of non-contradiction does not lead to a collapse into classical propositional logic. Additionally, I will demonstrate that extending any of da Costa's calculi C_n, where $1 \leq n < \omega$, with the theses of Aristotle or Boethius results in a trivial logic. To address this issue, I will define da Costa-like hierarchies of paraconsistent calculi, which, when extended by Aristotle's theses, will maintain their non-triviality. Unlike connexive logic, I will accept *consequentia mirabilis* as a thesis for all da Costa-like calculi. A noteworthy characteristic of these calculi is their negation inconsistency, which implies that there exists a formula α such that $\alpha \wedge {\sim}\alpha$ is a thesis of da Costa-like calculi.

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New Single Axioms for the Equivalential Calculus

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Tags: philosophical logic, proof theory.

One of the most important questions in the area of the equivalential calculus (EC) currently is the question of the single shortest axiom which is D-complete. The first single D-complete axiom for EC was showed by Ulrich [2] in 2005. In the paper [1] from 2024 I presented new single axioms for EC with the condensed detachment rule and the reversed condensed detachment rule that form D-complete bases and are organic. All of the axioms are 19-characters long. The question whether exist a shorter single axiom is still open.

I have recently found some new axioms for EC with the two rules. The new results bring us closer to answering whether there is a shorter single axiom of 19-characters. The new axioms are organic or inorganic as well. They constitute the base for EC with either the condensed detachment rule or the reversed condensed detachment rule. I know also formula which is an single axiom for EC but only in case where the two rules are avalaible.

List of new axioms is here:

$$EsEsEsEsEEEpqrEqErp$$
$$EEEEEEEpqrEqErpssss$$
$$EEEpqrEsEsEsEsEqErp$$
$$EEEEEEqEprssssErEqp$$

The new results and the proofs will be discussed. It will be showed also some interesting list of two-elements sets of axioms for EC that are very useful for searching a new results in the EC. It will be presented still open questions within the EC as well.

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Translation of Orthomodular Quantum Logic Into Modal Logic B

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Tags: modal logic, quantum logic.

The standard quantum logic based on non-distributive orthomodular lattices has been introduced in 1936 by George Birkhoff and John von Neumann in their seminal paper *The logic of quantum mechanics* [1]. Its algebraic semantics is given by orthomodular lattices, abstracted from the Hilbert space formalism of quantum mechanics. In the 1970s, several researchers have attempted to translate quantum logic into modal systems in order to interpret non-distributive logical operations using well-understood modal connectives.

In 1973, Robert Goldblatt formulated a representation theorem for ortholattices, which are the models of so-called *minimal quantum logic* [5]. Goldblatt proved that they are isomorphic to orthogonality spaces $\langle X, \perp \rangle$, where $\perp$ is a binary orthogonality relation on X. This result enabled the first modal semantics for a quantum logic, specifically the minimum quantum logic, which is not orthomodular. In 1974 Goldblatt proposed a translation of minimal quantum logic into modal logic B [6]. However, the translation of orthomodular quantum logic proved significantly more challenging—primarily due to Goldblatt's later result that the orthomodular law is not expressible in first-order logic [7]. There can be found in the literature many translations of orthomodular quantum logic to extensions of B, e.g., [6], [4], [3], [11] or T [2]. However, nowadays the orthoframe approach is by many considered unsatisfactory due to the lack of a simple expression of orthomodularity as a property of the relational frame [12].

In 2016 Chrysafis Hartonas proved a new representation theorem for orthomodular quantum logic and proposed a new relational semantics for orthomodular quantum logics. In this setting, orthomodularity becomes a first-order definable property by enriching the orthogonality space to a partially ordered orthoframe $F = \langle X, \perp, \leq, M \rangle$ [8]. Later, Joseph McDonald and Katalin Bimbó (2023) further developed this model by adding a topological component and proving a topological duality for orthomodular lattices [10].

The aim of this paper is to use this new semantic framework to obtain a translation of orthomodular quantum logic into the well-known modal logic B. The proposed approach draws on methods of formal and modal logic—including the translation techniques developed by McKinsey and Tarski [9]—to reinterpret the structure of non-distributive quantum logics in modal terms. As a result, this work fills a gap in the existing literature by offering a complete modal translation of orthomodular logic based on the relational semantics of $F = \langle X, \perp, \leq, M \rangle$ and by extending the range of modal interpretability of quantum logics.

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Towards a Logic of Rough Ternary Relations

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Tags: modal logic, semantics of modal logic, approximate reasoning.

1. Information systems

We intend to model affordances and actions in the framework of property and information systems in the sense of Vakarelov [6] and Pawlak [5]. A *property system* (*P-system*) is a structure $\langle U, V, f \rangle$, where U is a non-empty set whose elements are called *objects*, V is a set whose elements are called *properties*, and $f\colon U \to 2^V$ is a mapping called an *information function*; we do not require that $f(x) \neq \emptyset$. A statement $a \in f(u)$ can be interpreted as "Object u possesses property a". If U is finite, then a property system is definitionally equivalent to a formal dyadic context of Wille [7], by observing that the function $f\colon U \to 2^V$ can be replaced by a relation $R_f \subseteq U \times V$, where $u\ R_f\ a$ if and only if $a \in f(u)$ which has the same informational content.[7]

If we think of a property system as describing possible states of an attribute—such as "color" or "language spoken"—we extend it by the definition of an aggregate structure: An *attribute system* (*A-system*) (see [6]) is a structure $\mathcal{S} := \langle U, \Omega, \{V_a : a \in \Omega\}, f \rangle$ where

1. U is a non-empty set of objects,
2. Ω is a set of property labels or attributes, and V_a is a set of possible values of $a \in \Omega$,
3. $f\colon U \times \Omega \to \bigcup_{a\in\Omega} 2^{V_a}$ is a choice function, where $f(x, a) \subseteq V_a$. Equivalently, we may define $f : U \to \prod_{a\in\Omega} 2^{V_a}$.

So, if $a \in \Omega$ is a property label *weight*, then V_{weight} may be a set of rational numbers in some interval that can serve as numerical expressions of the weight of an object (e.g., in kilograms or pounds), or any value that makes sense. It could also be some aggregated value such as "low", "medium", "high" etc.

We call $\langle U, \Omega, \{V_a : a \in \Omega\} \rangle$ the *skeleton* of $\mathcal{S}$. The product $U \times \prod_{a\in\Omega} 2^{V_a}$ collects all possible vectors of value sets that can be associated with some element of U. An information function now picks one element from $\prod_{a\in\Omega} 2^{V_a}$ for each $x \in U$.

An element $x \in U$ is called *deterministic*, if $|f(x, a)| \leq 1$ for all $a \in \Omega$ or every projection of the vector attribute to x is either an empty set or a singleton subset of V_a. The set of all deterministic elements of $\mathcal{S}$ is denoted by $D_{\mathcal{S}}$. The characterization stems from the fact that the role of the choice function is to narrow down the possibilities for the values of a with respect to the object x. If $|f(x, a)| = 1$, then we know the exact value of the property a for x, or if $|f(x, a)| = \emptyset$, then we know that x does not have this

[7]At this stage of our investigation we suppose that we have a correct description of the world, i.e. what we observe is true.

property at all, or the value exists but it is uninteresting or unknown. This is why we call a system with $|f(x,a)| \leqslant 1$ deterministic. If $|f(x,a)| \geq 2$ the set has different possibilities of interpretation, see [3] and the references therein. If U is finite and $U = D_{\mathcal{S}}$, then $\mathcal{S}$ is called an *information system* (in the sense of Pawlak [5]).

2. Operationalizing affordances

A direction on operationalization of affordances was suggested in [2]:

> A formalization of affordance relations needs to provide crisp and fuzzy structures, mechanisms for spatial and temporal change, as well as contextual modeling.

The basic setup of an affordance relation consists of a set U of an agent's abilities, a set E of features of the environment, and a binary relation $R \subseteq U \times E$. Chemero [1, p. 189] writes

> Affordances [...] are relations between the abilities of organisms and features of the environment. Affordances, that is, have the structure Affords–φ (feature,ability).

We expand this notion by regarding an affordance in a first step as a relation $\varphi \subseteq A \times O \times E$ between actors, objects and properties of the environment, where $\varphi(a,o,e)$ is interpreted as

> Entity o affords action $\texttt{Act}_\varphi$ to the actor (or perceiver, agent) a in the environment (context) e.

The initial notion of an affordance is quite coarse, and all three components require further description. Therefore, we extend the concept as follows: Suppose that for a set A of actors, a set O of entities or objects, and a set E of environmental factors we have deterministic information systems

$$\mathcal{I}_A = \left\langle A, \Omega_A, \{V_q^A : q \in \Omega_A\}, f_A : A \to \textstyle\prod_{q\in\Omega_A} V_q^A \right\rangle,$$

$$\mathcal{I}_O = \left\langle O, \Omega_O, \{V_q^O : q \in \Omega_O\}, f_O : O \to \textstyle\prod_{q\in\Omega_A} V_q^O \right\rangle,$$

$$\mathcal{I}_E = \left\langle E, \Omega_E, \{V_q^A : q \in \Omega_E\}, f_E : E \to \textstyle\prod_{q\in\Omega_A} V_q^E \right\rangle.$$

Each of these information systems is interpreted as a description, respectively, of actors, entities, or the environment. We now define an *affordance* as a relation

$$\varphi \subseteq \{\langle a, f_A(a)\rangle : a \in A\} \times \{\langle o, f_O(o)\rangle : o \in O\} \times \{\langle e, f_E(e)\rangle : e \in E\}.$$

Thus, an affordance is a ternary relation that holds among actors with properties, objects with properties, and environments (contexts) with properties. See Figure 4 for a pictorial interpretation.

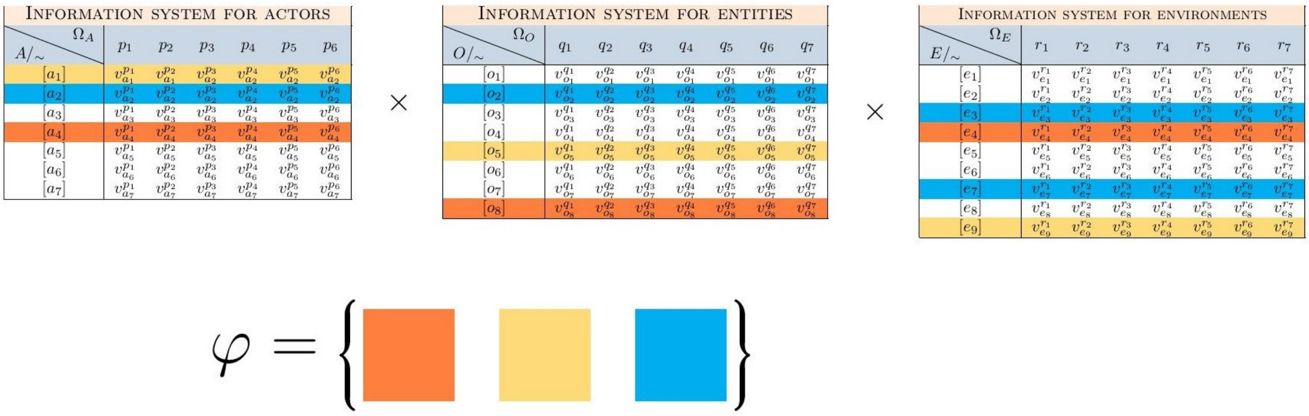

Figure 4: The triples of vectors of the same color constitute the affordance φ and its corresponding action $\texttt{Act}_\varphi$. We identify, respectively, actors, objects, and environments that cannot be distinguished by available properties.

3. Binary operators

We may extend the idea of a rough set [5] to relations. Given an affordance φ, we will abuse the notation writing $([a]\times[b]\times[c])\cap\varphi \neq \emptyset$ to say that there is $\langle x, y, z\rangle \in [a]\times[b]\times[c]$ such that

$$\langle\langle x, f_A(x)\rangle, \langle y, f_O(y)\rangle, \langle z, f_E(z)\rangle\rangle \in \varphi\,.$$

The analogous meaning is ascribed to the following statement:

$$[a]\times[b]\times[c] \subseteq \varphi\,.$$

With these, we can put a rough structure on affordances in a natural way. Firstly,

$$\langle\langle a, f_A(a)\rangle, \langle o, f_O(a)\rangle, \langle e, f_E(e)\rangle\rangle \in \overline{\varphi} \iff ([a]\times[o]\times[e])\cap\varphi \neq \emptyset\,.$$

Secondly,

$$\langle\langle a, f_A(a)\rangle, \langle o, f_O(a)\rangle, \langle e, f_E(e)\rangle\rangle \in \underline{\varphi} \iff [a]\times[o]\times[e] \subseteq \varphi\,.$$

In this way, we define the upper and the lower approximation of the affordance φ.

Definition. A *rough affordance* is a pair $\langle\underline{\varphi}, \overline{\varphi}\rangle$ such that φ is an affordance relation.

With any of the three relations, $\underline{\varphi}$, φ and $\overline{\varphi}$ we can associate modal-style binary operators, out of which *possibility* and *sufficiency* will be most important for us. Moreover, by means of φ we can define lower and upper approximation operators. We aim to show how these operators can be interpreted in the affordance setting, and how they can be used to incorporate roughness of reasoning into the formal framework.

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Incorporating Illocutionary Force Into Procedural Semantics Within Transparent Intensional Logic

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Tags: logic and artificial intelligence, philosophy of logic, philosophy of language.

Recently, there has been renewed discussion challenging the issue known as the "Frege–Geach Point", which demands that the same proposition can be affirmed and not affirmed at the same time, for example, when it occurs as a premise in a hypothetical argument (see [4]). Hanks, Recanati, and Bronzo [5, 6, 2] express objections to this. They argue that propositions or contents of sentences incorporate some kind of illocutionary force (e.g., assertive or directive), so that even though they share the same content, they count as different propositions if they have different force.

Transparent Intentional Logic (TIL) is a hyperintensional typed λ-calculus, that is a logical system, introduced by Pavel Tichý [1]. He claims that the syntactic difference between sentences with the same content does not reflect any difference in their logic. They express the same proposition, so they are represented by the same type. Duží, Jespersen and Materna [3] introduce Procedural Semantics within the context of TIL .

At this point, I raise a research question: could we further develop Procedural Semantics within TIL to increase the level of granularity in distinguishing between propositions that share the same logical/semantic content but differ in force? In other words, could such a conception of the proposition provide the latter with the capability of incorporating illocutionary force? I will attempt to address this question in a manner that is logically satisfactory.

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Leśniewski's Legacy and Grzegorczyk's Interpretation of Mereology: A Critical Reassessment

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Tags: history of logic, history of Polish logic, set theory and the foundations of mathematics, mereology.

Mereology, a formal theory of parthood developed by Stanisław Leśniewski in the early 20th century [4], has been the subject of logical investigations by various logicians of the Lvov-Warsaw School since its origins. In particular, in [1], Andrzej Grzegorczyk claims that Leśniewski's theory is essentially "the theory of a Boolean algebra of elements different from zero". To justify this claim, he constructs two elementary theories: *the elementary theory of Boolean algebra* and *the elementary mereology* – which he regards as Leśniewski's Mereology reformulated as an elementary theory – and argues that the axioms of the latter can be derived from those of the former, provided that the primitive terms of elementary mereology are appropriately defined within the language of the elementary theory of Boolean algebra. Grzegorczyk's constructions were later analyzed by Andrzej Pietruszczak in [2, 3], who criticizes them, arguing that certain features of these theories render the names assigned to them by Grzegorczyk inaccurate, as demonstrated by an analysis of their models.

In this presentation, I aim to retrace the development of Leśniewski's Mereology, its critiques, and various interpretations up to the present day, and to present some new results concerning certain unexpected models of Grzegorczyk's theories, which differ significantly from the intended ones and appear incompatible with Grzegorczyk's original intentions.

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Is the Overweighting of Low Probabilities Irrational?

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Tags: general methodology of science, philosophy of science.

Kahneman and Tversky, in their Prospect Theory (1979), maintain, among other things, that people, in the process of decision-making, tend to overweight low probabilities. In their account, this regularity—being at odds with the Maximization of Expected Utility—is of a purely psychological character. Laboratory findings by the Israeli psychologists have been confirmed by empirical studies of economic behavior, which point to the irrationality of the tendency in question. In contrast to such psychological explanations, I offer an account that explores a mathematical understanding of the very concept of probability. I argue that any probability distribution inevitably depends on certain assumptions about the realities under consideration. Since such assumptions are fallible, any given probability distribution may, in due course, be replaced with another. This possibility can be rationally taken into account in the process of decision-making. Thus, an alleged overweighting of low probabilities can, in many contexts, be explained as an anticipation of a possible future revision of the default probability distribution.

Varieties of Modal Algebras Without the Congruence Extension Property

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Tags: algebraic logic, modal logic.

The standard propositional modal language is given by a countably infinite set of propositional letters P and the usual logical connectives $\wedge$, $\neg$, $\bot$, and $\Diamond$. Other connectives such as $\top$, $\Box$, $\to$, $\vee$, etc. are taken as defined in the familiar ways. Let $\mathbf{S}$ be a modal system, i.e. a set of proof rules and axiom schemes. For a formula ψ and a set of formulas Γ we write $\Gamma \vdash_{\mathbf{S}} \psi$ and say that ψ can be derived from Γ, if there is a Hilbert-style derivation of ψ that uses only the formulas of Γ, and the axioms and the proof rules of $\mathbf{S}$. The modal system $\mathbf{E}$ has any axiomatization of propositional calculus as its axiom schemes, and has the modus ponens (MP) and the congruentiality rule (RE) as its proof rules.

$$(RE) \qquad \frac{\varphi \leftrightarrow \psi}{\Box\,\varphi \leftrightarrow \Box\,\psi} \qquad\qquad (MP) \qquad \frac{\varphi, \varphi \to \psi}{\psi}$$

The set of $\vdash_{\mathbf{E}}$-derivable formulas E is referred to as the smallest *classical modal logic* or *congruential modal logic*, i.e. this is the smallest set of modal formulas closed under instances of propositional tautologies and the rules (RE) and (MP), cf. [4]. The consequence relation $\vdash_{\mathbf{S}}$ (resp. the system $\mathbf{S}$) is an *axiomatic extension* of $\vdash_{\mathbf{E}}$ (resp. the system $\mathbf{E}$), if there is a set of formulas Σ, such that

$$\Gamma \vdash_{\mathbf{S}} \psi \quad \text{if and only if} \quad \Gamma \cup \Sigma \vdash_{\mathbf{E}} \psi$$

for any set $\Gamma \cup \{\psi\}$ of formulas. This means that $\vdash_{\mathbf{S}}$ can be obtained by adding new axioms Σ to the system $\mathbf{E}$.

The recent paper [3] proved that there are continuum many axiomatic extensions of $\vdash_{\mathbf{E}}$ that do not admit the local deduction detachment theorem. These axiomatic extensions of $\vdash_{\mathbf{E}}$ are rather ad hoc and lack any of the properties often considered in the literature, such as transitivity, reflexivity, normality, etc. It remained open (and the question was raised explicitly during the Cracow Logic Conference 2023) whether there are monotonic, normal, etc. axiomatic extensions of $\vdash_{\mathbf{E}}$ without the local deduction detachment theorem. This paper settles this question, improving on the results of [3]. In the talk we present the main results and techniques.

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Modal Validities: A Tool for Model Theory

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Tags: modal logic, model theory, semantics of modal logic.

Standard model theory concerns a given first-order theory T and investigates its models in order to better understand its nature. Research in modal model theory has the same goal, but tries to achieve it by studying T with the tools of modal logic. In contrast, modal model theory typically concerns itself with incomplete theories.

In this talk we will present some results concerning the investigations on the propositional frame $(Mod(T), \subseteq)$. We will provide some general results concerning intermediate logics and some tools for providing completeness. Furthermore, we will focus on the case where T is lattice theory or some stronger theory in the same language, the universe—all its models, and the accessibility relation is the relation of being an extension.

See [4, 6, 7, 9].

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Playing to Learn, or to Keep Secret: Alternating-Time Logic Meets Information Theory

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Tags: automated reasoning and automated theorem proving, epistemic logic, logic and artificial intelligence.

Many important properties of multi-agent systems refer to the participants' ability to achieve a given goal, or to prevent the system from an undesirable event. Among intelligent agents, the goals are often of epistemic nature, i.e., concern the ability to obtain knowledge about an important fact φ. Such properties can be, e.g., expressed in ATLK, that is, alternating-time temporal logic ATL extended with epistemic operators. In many realistic scenarios, however, players do not need to fully learn the truth value of φ. They may be almost as well off by gaining *some* knowledge; in other words, by reducing their uncertainty about φ. Similarly, in order to keep φ secret, it is often insufficient that the intruder never fully learns its truth value. Instead, one needs to require that his uncertainty about φ never drops below a reasonable threshold.

With this motivation in mind, we introduce the logic ATLH, extending ATL with quantitative modalities based on the Hartley measure of uncertainty. The new logic enables to specify agents' abilities w.r.t. the uncertainty of a given player about a given set of statements. It turns out that ATLH has the same expressivity and model checking complexity as ATLK. However, the new logic is exponentially more succinct than ATLK, which is the main technical result in this talk.

The research have been published in the conference paper [1].

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Suszko-Style RNmatrix Reductions: Truth-Functionality Lost

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Tags: many-valued logic, non-classical logic, philosophical logic, philosophy of logic.

In this talk, we give a novel formalization of what we call Suszko-style reductions, using the machinery of restricted non-deterministic matrices (RNmatrices), as developed by Coniglio and Toledo in [3] and anticipated by, e.g., Kearns [6] or Omori and Skurt [8]. Suszko-style reductions are generalizations of the original Suszko's Reduction, i.e., the proof that every Tarskian consequence relation can be characterized by a two-valued semantics. Analogous (but many-valued) reductive results have since continued to develop for weaker, non-Tarskian logics.

Roman Suszko's result led to the controversial philosophical claim that, in effect, "every logic is [...] two-valued" ([9], p. 378), which has become known as Suszko's Thesis and has been subject to much criticism. One important point of criticism is that the resulting semantics is not always truth-functional. As Suszko says, "[e]ach adequate set of logical valuations bears a natural (dual) pre-topology which turns out to be a genuine topology in certain important cases." ([9], p. 378) In other important cases, however, it turns out not to be one.

More importantly, however, (non-truth-functional) many-valued semantics can be obtained using Suszko-style reductions for some non-Tarskian logics. This was first done by Grzegorz Malinowski in [7], who proved that every monotonic reflexive logic (q-consequence) has an at most three-valued semantics. An analogous result may be obtained for its dual, monotonic transitive logic (formalized by Szymon Frankowski as p-consequence in [4]). Moreover, recently, it has been proved in several settings that every monotonic logic has an at most four-valued semantics (by French and Ripley [5], Blasio, Marcos and Wansing [1], and Chemla and Égré [2]).

Following some remarks of Coniglio and Toledo's [3] as implicitly restating Suszko's Reduction in terms of their RNmatrix semantics, we propose that this result be extended to allow for a generalization in two directions. First, we generalize to the multiple-conclusion framework of Scott consequence relations, as opposed to the one-conclusion Tarskian logics. Second, we modify the RNmatrices to account for some weaker logics. The main formal results to be discussed are as follows:

Theorem 1. *Every Scott consequence relation is characterized by a two-valued RNmatrix.*

Theorem 2. *Every monotonic reflexive logic (p-consequence) is characterized by a three-valued RNmatrix.*

Theorem 3. *Every monotonic transitive logic (q-consequence) is characterized by a three-valued RNmatrix.*

Theorem 4. *Every purely monotonic logic is characterized by a four-valued RNmatrix.*

Our account of the Suszko-style phenomena has several notable advantages. As stated above, it not only generalizes to multiple conclusions but also provides a uniform treatment of all the reductions in tandem. Moreover, due to the use of non-deterministic matrices, it offers a natural explanation of the loss of truth-functionality obtained by the reductions.

The resulting RNmatrix semantics is straightforward and elegant, requires no extensive detours into model theory, and, more importantly, it is the only method known to us that offers a matrix semantics for the Suszko-style reductions. Philosophically, due to this observation, we propose that these results lead to some significant challenges for the use of RNmatrices as a perspicuous semantics.

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STV+KH: Towards Practical Verification of Strategic Ability for Knowledge and Information Flow

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Tags: epistemic logic, modal logic, temporal logic.

1. Introduction

Model checking of multi-agent systems allows for formal verification of their relevant properties. An important group of such properties concerns the ability of agents to achieve (or prevent) a given state of affairs [2, 20, 5, 18]. This is often combined with requirements regarding the agents' knowledge or uncertainty. For example, one can address the ability of a voter v to eventually know whether her vote has been registered correctly for candidate j (*voter-verifiability*), or to maintain the coercer's uncertainty about the value of the vote (*vote privacy*). The former requirement can be specified in the epistemic extension of alternating-time temporal logic **ATLK** [22, 11] by the formula $\langle\langle v\rangle\rangle \mathrm{F}(K_v \mathsf{vote}_{v,j} \vee K_v \neg\mathsf{vote}_{v,j})$. The latter is captured in **ATLH** by $\langle\langle v\rangle\rangle \mathrm{G} H_c^{\geq k}(\mathsf{vote}_{v,1}, \ldots, \mathsf{vote}_{v,n})$, where H is the uncertainty modality proposed recently in [21], n is the number of candidates in the election, and k says how many bits of uncertainty we want on the side of the coercer. Here, we propose a new extension of our experimental tool **STV** [13, 15], that allows to verify such specifications for models of asynchronous MAS.

Related work. A number of model checkers for agent logics have been proposed over the last 25 years. Of those, MCK [9] addresses only epistemic properties; Mocha [1], the PRISM family [6], STV [13, 15], and most Strategy Logic extensions of MCMAS [4] admit only strategic-temporal operators. MCMAS [16] and MCMAS-SLK [3] allow for strategic and epistemic modalities, but concentrate on perfect information strategies. Our new proposal, **STV+KH**, combines verification of memoryless imperfect information strategies with specifications of agents' knowledge. Even more importantly, it allows for the analysis of how agents can influence the information flow and the quantitative uncertainty in the system.

Application domain. **STV+KH** addresses formal verification of MAS, which is a nontrivial problem [7]. Anonymity, privacy, and effective information exchange are essential requirements for many systems. **STV+KH** offers a user-friendly environment for the analysis of such requirements, including a GUI and a flexible model specification language. Moreover, **STV+KH** has a strong pedagogical valor, as it can be used for an intuitive introduction to the complicated subject of strategic reasoning and model checking of strategic logics. The previous versions of **STV** have already been used in tutorials and graduate courses at IJCAI, PRIMA, and ESSAI.

2. Formal Background

Modules. The main part of the input is given by a set of asynchronous modules [17, 10], where local states are labelled with valuations of state variables. The transitions are valuations of input variables controlled by the other modules. The global model of the MAS is defined by the asynchronous product of its modules.

Strategies. A strategy is a conditional plan that specifies what the agent(s) are going to do in every possible situation [2, 20]. Here, we consider the case of *imperfect information memoryless strategies*, represented by functions from the agent's local states to its available actions. The *outcome* of a strategy from state q consists of all the infinite paths starting from q and consistent with the strategy.

Logic. Given a model M and a state q in the model, the formula $\langle\langle A\rangle\rangle\varphi$ holds in M, q iff there exists a strategy for agents A that makes φ true on all the outcome paths starting from any state indistinguishable from q [2, 20]. Moreover, $K_a\varphi$ holds in M, q iff φ is true in every state q' indistinguishable from q for a [8]. Finally, $H_a^{\leq r}(\varphi_1, \ldots, \varphi_n)$ holds iff the number of possible valuations for $(\varphi_1, \ldots, \varphi_n)$ in a's indistinguishability class can be represented on r bits. I.e., the Hartley measure of uncertainty for a is at most r [21].

Example scenario. As a working example, we use the Asynchronous Simple Voting scenario [12]. The model consists of k voters and a single coercer. Figure 5 presents the global model with one voter. There are several propositional variables in the model: $\mathsf{vote}_{i,j}$: whether the voter i has voted for the candidate j; pun_i: whether the voter i was punished or not; finish_i: whether the voter i has finished the voting process and her interactions with the coercer. The voter first casts her vote, then decides whether to share its value with the coercer. Finally, she waits for the coercer's decision to punish her or to refrain from punishment. The coercer has two available actions per voter: to punish (or not) the voter.

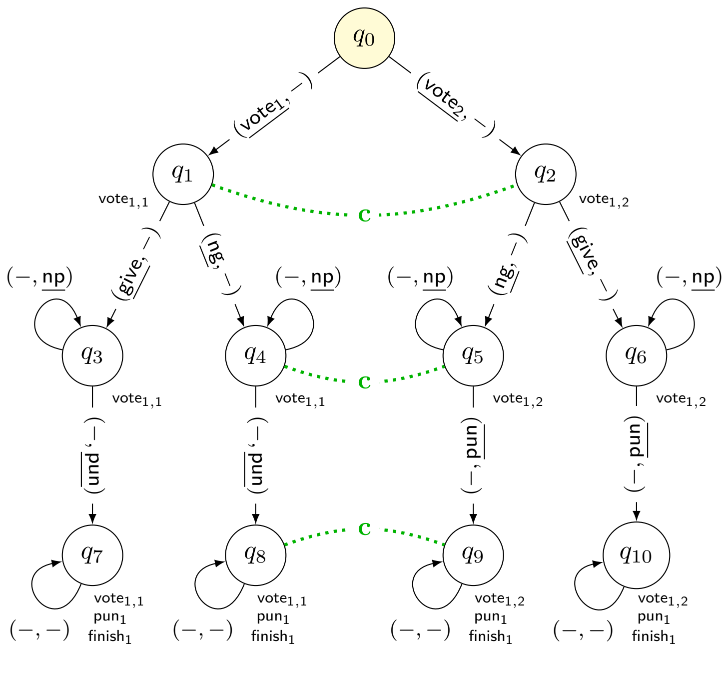

Figure 5: Simple voting model with 1 voter and 2 candidates.

3. Technology and Usage

STV+KH does *explicit-state model checking.* That is, the global states and transitions of the model are represented explicitly in the memory of the verification process. The user can load and parse the input specification from a text file that defines the modules, i.e., local automata representing the agents. The generated models and the verification results are visualised in an intuitive web-based graphical interface. The verification algorithms are implemented in C++, and the GUI in Typescript, using the Angular framework.

The tool is available at stv.cs-htiew.com. The video demonstration of the tool is available at youtu.be/yDuK_Vsr8sQ. Example specifications can be found at stv-docs.cs-htiew.com. The current version of **STV+KH** allows to: generate and display the composition of a set of modules into the model of a multi-agent system; provide local specifications for modules, and compute the global specification as their conjunction; verify an **ATLK** and/or **ATLH** reachability or safety formula with knowledge and/or uncertainty operators (nested strategic operators are not allowed); display the verification result including the relevant truth values.

4. Experimental Evaluation

We evaluate the performance of the new operators on two benchmarks: the Simple Voting example from Section 2, and the much more sophisticated family of models for the voting protocol Selene [19, 14]. All times are given in seconds. The timeout was set to $3h$. The test platform was a server with ninety-six 2.40 GHz Intel Xeon Platinum 8260 CPUs, 991 GB RAM, and 64-bit Linux.

Simple Voting. For Simple Voting, we used the **ATLK** formula:

$$\phi_1 \equiv \langle\langle c \rangle\rangle \mathrm{G}((\mathsf{finish} \wedge \mathsf{vote}_{i,j}) \rightarrow K_c \mathsf{vote}_{i,j})$$

Thus, ϕ_1 expresses the undesirable property of *strategic anonymity breach*, saying that the coercer can ensure that the coercer knows the value of i's vote whenever the election comes to an end and voter i has voted for candidate j. We use $i = j = 1$ in all experiments. The experimental results are shown in Table 1. **STV+KH** was able to verify up to 6 agents (5 voters and 1 coercer). The formula was *false* in all instances, i.e., no anonymity breach was found.

#V	States	Time	Result
1	15	0.005	False
2	133	0.008	False
3	1071	0.097	False
4	8461	1.559	False
5	66855	52.493	False
6	timeout		

Table 1: Results for Simple Voting: 2 candidates, formula ϕ_1

Selene. For Selene, we first verified ϕ_1, showing that the coercer cannot gain exact knowledge about the voter's vote also in that case (see the results in Table 2). Then, we verified the **ATLH** formulas

$$\phi_2 \equiv \langle\langle c \rangle\rangle \mathrm{G}(\mathsf{finish} \rightarrow H_c^{\leq 2}(\mathsf{vote}_{i,1},\ \mathsf{vote}_{i,2}, \ldots,\ \mathsf{vote}_{i,n})),$$

which turned to be true in all instances, and

$$\phi_3 \equiv \langle\langle c \rangle\rangle \mathrm{G}(\mathsf{finish} \rightarrow H_c^{\leq 1}(\mathsf{vote}_{i,1},\ \mathsf{vote}_{i,2}, \ldots,\ \mathsf{vote}_{i,n})),$$

which was always false. Thus, the coercer can reduce his uncertainty about the voter's vote to at most 2 bits, but not further down to 1 bit. Throughout the experiments, we used $i = 1$ and $n = 3$. We were able to verify up to 4 agents (3 voters and 1 coercer).

#V	**States**	ϕ_1		ϕ_2		ϕ_3	
		Time	**Res.**	**Time**	**Res.**	**Time**	**Res.**
1	1267	0.1	False	0.1	True	0.1	False
2	38530	2.5	False	2.7	True	2.6	False
3	2195950	180.7	False	200.4	True	184.3	False
4	timeout						

Table 2: Results for Selene with 3 candidates

5. Conclusions

We present **STV+KH**: a substantial extension of the **STV** model checker, augmented with modalities for agents' knowledge and quantitative uncertainty. The experiments show that the verification of anonymity-related properties using **STV+KH** performs similarly to model checking "vanilla" strategic properties, reported in [13, 15, 14]. Thus, we gain significant expressivity with little price in terms of complexity and performance.

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On the Idea of Philosophical Duality Between Formal Systems

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Tags: philosophy of logic, paraconsistent logic, formal theories of truth.

The aim of this talk is to provide an overview of a novel concept of philosophical duality. Intuitively, it is a property exhibited by formal systems where one's philosophical interpretation can be rewritten in a dual manner, thereby providing a philosophical interpretation for the other. The analysis draws on observations from a methodology of formal philosophy [1]; the history of paraconsistent logic, especially the Jaśkowski's Criterion [2]; and recent work on the notion of philosophical interpretation of formal systems [3, 4].

To illustrate the usefulness of the coined notion, a case study of a particular problem in the domain of paraconsistent (formal) theories of truth is considered, namely whether one can provide a philosophically robust interpretation of paraconsistent such theories that would be commensurate with interpretations of paracomplete theories studied in the literature [5, 6]. The breaking of the philosophical duality in the context at hand is demonstrated and some philosophical conclusions regarding future work are drawn.

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Definite Descriptions in Free Core Logic

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Tags: non-classical logic, philosophical logic, proof theory.

When formulated in natural deduction, in Neil Tennant's Core Logic [1] deductions are by definition in normal form, that is to say, major premises of elimination rules "stand proud" and are (discharged or undischarged) assumptions of the deduction, not conclusions of any rules of inference. A challenge for the Core Logician is to show that a deduction that concludes with a complex formula and a deduction in which this formula is the major premise of an elimination rule can be combined into a deduction of Core Logic. The process is comparable to normalisation in intuitionistic logic, but amounts to something more like cut admissibility in a cut free sequent calculus. In fact, cut is not a primitive rule of the sequent calculus version of Core Logic, but a version of it can be shown to be admissible. Instead of maximal formulas, I shall therefore speak of cut formulas when it comes to Core Logic.

Tennant has recently proposed a negative free version of core logic, in which singular terms are admitted that do not refer and the domain of quantification may be empty. An important aspect of the development of free logic by Hintikka, Lambert and others was to formalise theories of definite descriptions that are alternatives to Russell's. Tennant, too, considers adding the ι operator for definite descriptions to negative free core logic. This was done recently in a talk at the ExtenDD seminar of Andrzej Indrzejczak's ERC Grant Coming to Terms (`https://www.uni.lodz.pl/fileadmin/Projekty/EXTENDD/Neil_Tennant_talk.pdf`).

The aim of the present talk is threefold: first, to demonstrate how a certain problem that arises with normalisation of intuitionistic negative free logic with ι does not arise in Negative Free Core Logic; secondly, to show how to formalise a positive version of Free Core Logic with ι; thirdly, to consider an alternative approach to the formalisation of definite descriptions by a binary quantifier in free Core Logic.

The problem, in a nutshell, is this. With negative free logic formalised as it usually is, in normalisation a problem arises with the reduction procedures for the quantifiers and the ι operator. Parameters, which are a singular term of no complexity, are substituted by arbitrary terms which, in the presence of ι, may have an arbitrary complexity. This may introduce maximal formulas of unknown degree into a deduction as the result of the reduction procedure.

The problem has been solved by transposing a technique introduced by Indrzejczak [3] in the context of sequent calculus to natural deduction [2]: replace the usual introduction rule for $=$ ($\exists! t \vdash t = t$) by an alternative proposed by Indrzejczak (if $\Gamma, a = t \vdash C$, where a is a parameter that does not occur in Γ, t, C, then $\Gamma, \exists! t \vdash C$).

I shall demonstrate that, remarkably, Indrzejczak's trick is not needed in Negative Free Core Logic. This is because by definition, deductions in Core Logic are by definition in normal form, so replacing a parameter by a complex term cannot introduce any maximal formulas of unknown degree either. The procedure that establishes the admissibility of cut is, however, rather intricate. New cut formulas may by introduced by the procedure that turns two deductions into a new deduction of Core Logic when the cut formula is an identity flanked by an ι term, any increase in the complexity of the cut formula relative to the one removed is under control, as a complex term occurring in a premise of an ι rule is replaced for a parameter in a subformula of the cut formula: the exact increase in the complexity is therefore known, and any problem removed by adding the degree of the term to the degree of the cut formula in the measure of the complexity of the deduction. Furthermore, the induction procedes, unusually for natural deduction, over a measure that involves the degree of the cut formula and the maximum of the heights of the two deductions to be transformed into a new deduction.

The hard work having been done I shall round things off by indicating how to formalise a positive free version of Core Logic without and with definite descriptions and how to add a binary quantifier for the formalisation of definite description in the spirit of Russell in the context of complete sentences 'The F is G' to negative and also to positive Free Core Logic. In all these cases, it is essential to indicate how procedure of establishing cut admissibility works, but it is similar to that of Negative Free Core Logic.

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What Does a Sentence Denote? Zdzisław Dywan's Logic for Aristotle's and Frege's Concept of Denotation

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Tags: history of Polish logic, philosophy of logic.

In his work *Denotation in Aristotle and Frege*, Zdzisław Dywan analyses the Megarian paradox of The Horned Man. He points out that the source of the paradox of The Horned Man is the confusion of two different concepts of denotation—Aristotle's, according to which every sentence is either true or false, and Frege's, who divides sentences into those that have denotations and those that do not, and then divides those that have denotations into true and false.

In my paper, I will present Dywan's analysis of the two concepts of denotation and place it in the broader context of the discussion (in particular by Suszko and Prior) on the so-called Frege's axiom and the intuitive difficulties arising from it.

See [1, 2, 3, 4].

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What's Between Questions? Erotetic Implication, Interpolation, and Erotetic Rationality

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Tags: logic of questions, philosophical logic, philosophy of science.

In recent years, the topic of interrogative epistemic rationality has (once again) come to the fore [12, 1, 2, 3, 4, 5, 9, 10]. Erotetic implication, introduced by Andrzej Wiśniewski within the paradigm of Inferential Erotetic Logic [11, 13, 14], has been applied successfully in the philosophy of science [6, 7], and is being discussed as a formal explication of a norm of erotetic rationality [9, 10].

A recurring claim among those specializing in the subject, sometimes expressed in the literature [6, p. 95–97], is that erotetic implication is too strong, since some interesting cases of erotetic reasoning do not meet all of its criteria. In my talk, I will argue the opposite. I will present the Interpolation Theorem for Questions [8], which shows that in many significant cases of inferring a question Q^* from a question Q and declarative premises X, it suffices to satisfy just one criterion of erotetic implication: namely, that the soundness of Q together with the truth of the sentences in X is transmitted into the soundness of Q^*. If this condition holds, one can construct an auxiliary question $Q \odot Q^*$, which is implied by Q and in turn implies Q^*. This auxiliary question can be viewed as an enthymematic erotetic premise, the unveiling of which restores the soundness of the whole reasoning.

I will sketch the proof of the theorem and discuss its consequences. The theorem reinforces the view that erotetic implication functions as a normative yardstick for erotetic inferences, and thus provides a basis for formulating erotetic norms of rationality.

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Legal Logic in the Age of AI

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Tags: legal logic, logic and artificial intelligence, general methodology of sciences.

The paper discusses the scope of legal logic tailored to the needs of contemporary legal practitioners. The author, a legal practitioner with over thirty years of professional experience, and at the same time a trained logician, evaluates the changes that have occurred over the past few decades in the practice of law, in the context of the widespread use of computers, the Internet, and more recently AI. The goal is to propose a broader framework for the discipline referred to as 'legal logic'. The author argues that systematically developing this proposed scope should be a joint effort of legal practitioners and logicians who study the properties of argumentation and communication, including the applications of artificial intelligence. A broader incorporation of logical methodology and AI knowledge could significantly contribute to the improvement of legal education models, and consequently, to better law enforcement.

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Toward a Logic of Conceptual Change

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Tags: epistemic logic, philosophical logic, philosophy of science.

Traditional accounts of abduction characterize it as inference to the best explanation, where an agent selects the hypothesis that most adequately accounts for observed data relative to a fixed background theory [1, 2]. Such first-order abduction operates within an established conceptual scheme and presupposes stable evaluative standards. However, cases of radical theory change—paradigmatic scientific revolutions in the Kuhnian sense [3]—require a more robust form of reasoning that transcends the confines of any single conceptual framework.

I propose to characterize this more fundamental mode of reasoning as second-order abduction, wherein agents evaluate and compare entire conceptual frameworks, rather than individual hypotheses internal to them. This process involves assessing which framework offers the most coherent, comprehensive, and explanatorily powerful account of recalcitrant phenomena. Crucially, such assessment cannot be reduced to first-order abductive moves, as it involves meta-level considerations about the standards of adequacy themselves.

I suggest that second-order abduction can be formally understood as a form of counterfactual modeling over conceptual frameworks. Rather than merely asking which hypothesis best explains the data, agents engage in counterfactual evaluation: What if we adopted an alternative framework? Would the anomalous data be better accommodated? This counterfactual dimension enables agents to consider possible conceptual "worlds" in which explanatory success is maximized, thereby facilitating rational conceptual change.

To articulate this formally, I propose integrating counterfactual logics [5, 4] with dynamic epistemic logic [6], thereby modeling conceptual frameworks as structures over which counterfactual conditionals can be evaluated. In this schema, transitions between frameworks can be captured via dynamic updates, permitting precise representation of both propositional content change and shifts in inferential standards. Further, connections with iterated belief revision [7, 8, 9] illuminate how agents can rationally revise their higher-order commitments in response to persistent anomalies.

Beyond its significance for philosophy of science, this account bears directly on current discussions in epistemic logic and the logic of AI systems. In particular, meta-learning and model selection tasks in artificial intelligence exemplify processes akin to second-order abduction, wherein systems must select or revise representational schemas under shifting informational constraints. By formalizing second-order abduction through counterfactual modeling, we gain new tools for analyzing the epistemic structure of scientific revolutions and for designing more interpretable and adaptable artificial agents.

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Logical Culture in the Age of AI: Reviving Ajdukiewicz's Educational Vision

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Tags: history of Polish logic, logic and artificial intelligence, logic education.

In an age marked by overwhelming information, polarizing discourse, and the rapid evolution of technologies like generative AI, students increasingly turn to large language models (LLMs) for answers—not only in routine tasks but also in areas that require conceptual clarity, accuracy, and genuine understanding. Against this backdrop, we argue that logic education is more urgent than ever.

To clarify its contemporary mission, we turn to the Lvov–Warsaw School, and especially Kazimierz Ajdukiewicz, who urged that logic—broadly understood to include formal logic, semiotics, and the methodology of science—belongs at the core of general education [1]. What gives this tradition its present force is its insistence that logic is more than a set of technical rules. It is a formative discipline that fosters what Ajdukiewicz called *logical culture*—the habit of reasoning clearly and reflectively, sustained by a principled commitment to rigour and a willingness to question assumptions. That vision, we maintain, offers precisely the scaffolding needed to cultivate thinkers capable of navigating today's layered epistemic pressures—from viral arguments on social media to the syntactic fluency of LLMs.

Building on Ajdukiewicz's framework, the paper pursues three aims. First, it diagnoses the reasoning failures that LLMs reveal on inference tasks and shows how these lapses can be turned into teachable moments. Second, it contrasts rule-based intelligent tutoring systems, in particular Diderik Batens' "Logicaprogamma" from the 1980s (today known as the web-based *Alice*) with dialogue-oriented AI agents (such as *Personify*, [3, 2]), arguing that the two offer complementary pedagogical strengths. Third, it develops design principles for a blended curriculum that leverages these tools to sharpen—rather than dull—students' logical competence, positioning engagement with AI as a catalyst for more rigorous and reflective reasoning.

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Commands as Normative Announcements

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Tags: deontic logic and action, epistemic logic, semantics of modal logic.

Interpreting speech acts as operations over models has proven highly effective in Deontic Logic. The updated models are tipically preference models, in which a betterness ordering among possible worlds is either assumed or derived from a priority relation among propositions (See [2]). While dynamical interpretations based on preference models seem particularly well-suited for representing norms as "optimization commands" (in terms of [1]), rules, by contrast, apparently do not appear to require any underlying betterness relation in our models. In this presentation we explore the consequences of treating commands and authorisations as simple operations that reduce or enlarge the set of permitted states in $SD4$ models (see [5]) which contain no betterness order but instead rely on an ideality relation that is serial, transitive and secondary-reflexive (according to Chellas's terminology in [4]). The aim is to investigate both operations in the simplest framework to represent both types of imperatives uniformly—contrary to the positions held by some logicians that commands could be pictured as operations on models, while authorisations as the removal of a prohibitive norm from a set of norms [3]. Although such difference may not be reflected as a different operation from that of enlarging our models, seems to rely on the assumption that every state of affairs has a deontic nature—it is permitted—when such a removal takes place.

We also emphasise the importance of preserving the structural features of our models before and after updates, in order to maintain a fixed interpretation of obligations and permissions throughout. This concern leads to a deeper philosophical question: if commands and authorisations are the only operations available for modifying models, is it possible to view such operations as giving rise to *initial* deontic models as well? This could be framed as a foundational problem for normative systems. Can authorisations and commands, from a logical point of view, account for the emergence of such systems?

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A General Approach in Relational Semantics for Some Weak (Modal) Logics

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Tags: intuitionistic logic, philosophical logic, semantics of modal logic.

We present two types of relational structures which serve as semantic tools in studies on logics which require non-standard treatment of connectives or modal operators due to their hyperintensionality [2, 7]. We demonstrate how these structures, with appropriate definitions of truth, describe subnormal modal logics, including the system **N** known from [3] as well as its intuitionistic counterpart **NINT** and their extensions which form classes of classical and intuitionistic intermediate—between **N** and **K** (the weakest classical normal modal logic), and between **NINT** and **KINT**—subnormal modal logics [5]. Furthermore, introduced structures provide semantics for a paraconsistent logic **CluN** [1], thereby allowing for a translation between **CluN** and the subnormal modal logic axiomatised by $A \to \Box A$. Finally, the presented structures provide relational semantics for a 'failed axiomatisation' of **K**—a problem noted in [4]—and its extensions for which the only semantic theory known so far was non-deterministic semantics devised in [6].

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Absolute Arithmetic vs Church's Thesis and the Consistency of the PA

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Tags: logic and the foundations of computer science, philosophy of logic.

Absolute arithmetic is a set of properties and methods relating to the quantitative (computational) properties of the world and the human activity that realises computation. In this sense—similar to the Aristotelian category of quantity—absolute arithmetic cannot be conceived as yet another mathematical structure.

Moving into the mental realm—through abstraction from materiality (physicality) creates theories based on absolute arithmetic. Two important theories of this type are:

- Peano's arithmetic as a description of numbers and their properties;
- computability theory as a description of efficient computational methods.

Both of these theories have a common part (e.g., representability of recursive functions).

By introducing in an appropriate way the notion of adequacy of the theory with respect to absolute arithmetic, we can see Church's thesis as a confirmation of the adequacy of the theory of computability (cf. [1]). On the other hand, it can be argued that the adequacy of Peano's arithmetic requires the consistency of the theory.

As can be seen, the consistency of Peano's arithmetic and Church's thesis thus become strongly linked (a similar approach has—in a different context—some history, see [2]). The paper argues that the problem of the connection between Church's thesis and the consistency of arithmetic is a problem that requires a philosophical approach (rather than mathematical one). However, the consequences of philosophical considerations of this type have implications for mathematical theories. Moreover, under realistic philosophical assumptions, the interdependence between Church's thesis and the consistency of arithmetic can be demonstrated.

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On a Proposal of a Discussive-Like Logic and Its Relation to Discussive and Connexive Ones

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Tags: modal logic, non-classical logic, paraconsistent logic.

In this paper, we provide some logico-mathematical rudiments to develop discussive systems of connexive logics. We expand the modal connexive logic CK introduced in [6]. CK is characterized by the following axioms and rules:

(INT) All axioms schemes of intuitionistic positive logic.

(DNL) $\sim\sim A \leftrightarrow A$

(DM1) $\sim(A \vee B) \leftrightarrow (\sim A \wedge \sim B)$

(DM2) $\sim(A \wedge B) \leftrightarrow (\sim A \vee \sim B)$

(HC) $\sim(A \to B) \leftrightarrow (A \to \sim B)$

(K2) $\Box A \wedge \Box B \to \Box(A \wedge B)$

(L⊤) $\Box(A \to A)$

(M6) $\Diamond(A \vee B) \to \Diamond A \vee \Diamond B$

(K7) $\Diamond(A \to B) \to (\Box A \to \Diamond B)$

(KT□) $(\Diamond A \to \Box B) \to \Box(A \to B)$

(Df◇) $\sim\Box A \leftrightarrow \Diamond\sim A$

(Df□) $\sim\Diamond A \leftrightarrow \Box\sim A$

(K9) $\Box(A \vee B) \to \Box A \vee \Diamond B$

(MP) $A, A \to B/B$

($RM_\Box$) $A \to B/\Box A \to \Box B$

($RM_\Diamond$) $A \to B/\Diamond A \to \Diamond B$

We introduce CD, the connexive modal analogue of the deontic normal modal logic **D**. We obtain a completeness result for CD. Similarly as **D**, CD equals its ◇-counterpart. Based on CD we consider a discussive logic.

The motivation that we consider for the introduction of a new discussive logic is that, under certain intuitive conditions connexive logic requires a paraconsistent system of logic. There are some studies on connexive paraconsistent logics. A notable example is [4], where a connexive logic based on the paraconsistent logic **LP** was introduced. Some negative results have been proved by Ciuciura in [1] regarding the paraconsistent Brazilian tradition. Ciuciura proved that there cannot be non-trivial connexive extensions of da Costa's paraconsistent systems C_n, $1 \leq n < \omega$. There are, however, connexive extensions of a similar hierarchy based on an intutionistic implication. It is important to observe, however, that with the only exception of [2], there have been almost no studies of discussive systems, another important tradition in paraconsistent logic, for connexive logics.

Relying on the properties of the modal logic CD, we consider a proposal of a discussive logic based on CD, and provide an analysis of its relation to the traditional discussive logic and to connexive logics. In particular, we compare some of its properties with the discussive logic D_2, with a minimal variant of discussive logics given in [3], and with some connexive logics.

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Positional Logic as a Logic of Fiction

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Tags: non-classical logic, philosophical logic, formal ontology.

Since its inception, positional logic has been acknowledged as a highly versatile formal framework. It has demonstrated utility across diverse domains, including temporal and epistemic reasoning, as well as in the formal treatment of topological structures and possible worlds semantics. In this presentation, I aim to build upon Nicholas Rescher's interpretation of positional logic as a logic of possible worlds by employing an interpreted realization operator [1]. This approach serves as the foundation for formulating a preliminary framework for a logic of fictional objects.

To begin, I will introduce the fundamental concepts that underpin positional logic and outline the axioms of its elementary system known as the Minimal Realization Logic (formally denoted as **MR**). This logic centers around the realization operator $\mathcal{R}$, which associates the truth value of an expression, denoted by p, within a certain context, denoted by α. The resulting expression would have a following form $\mathcal{R}_\alpha p$. Such expression should be read as "a sentence p is realized at a context α".

The most basic system of positional logic was introduced in the work [2]. The language of this logic comprises propositional variables $p, q, r, ...$, positional letters $\alpha, \alpha_1, \alpha_2, ...$, standard logical connectives, and the realization operator $\mathcal{R}$. The axiom system of this logic is defined by the following formulas, which are closed under the *Modus Ponens* rule:

Axiom 1. $e(A)$, if A is a tautology of **CPL** and e is a mapping from the language of **CPL** to the language of **MR**.

Axiom 2. $\mathcal{R}_\alpha \neg A \leftrightarrow \neg \mathcal{R}_\alpha A$

Axiom 3. $\mathcal{R}_\alpha (A \wedge B) \rightarrow (\mathcal{R}_\alpha A \wedge \mathcal{R}_\alpha B)$

Axiom 4. $\mathcal{R}_\alpha A$, if A is a tautology of **CPL**.

Building upon the outlined axiomatic system and accompanying semantics, I will present a preliminary formalization of a neo-Meinongian theory utilizing the realization operator. Specifically, the focus will be on a variant of Modal Meinongianism as developed by Graham Priest and Richard Routley [5].

In this draft, the realization operator will play a role of a fiction operator, while positions, typically construed as contexts, will be interpreted as possible worlds. Under this interpretation, positional logic can be viewed as a logic of possible worlds. However, our objective extends beyond this general perspective: we aim to analyze the ontological status of possible objects, particularly with regard to their existence or nonexistence. This ontological dualism will serve as a central theme of the presentation.

In classical ontology, all objects are assumed to exist. Thus, entities such as possible, fictional, or even impossible objects are often dismissed as mere illusions of content. By contrast, Meinongian and neo-Meinongian approaches take such objects seriously, granting them a status comparable to that of spatio-temporal entities. This divergence stems from two foundational assumptions of the Meinongian framework.

The first assumption is that every intentional act is directed toward an object, regardless of whether that object exists. In this view, all thought is about some object, whether actual or non-actual. The second assumption distinguishes between 'being' and 'existence'. To be an object does not entail existence. This stands in opposition to the ontological commitments of thinkers such as Russell and Quine, who maintain that to be is to exist [5].

From a Meinongian perspective, 'being' is ontologically prior distinct from 'existence'. Objects may possess properties without necessarily existing. Consider the fictional character Gandalf the Grey. We attribute to him properties such as smoking a pipe, wearing a grey robe, and having a long beard. At the same time, we deny that he is a short, ill-tempered shopkeeper in London. These truth-value attributions in natural language suggest that statements like "Gandalf the Grey is clothed in a grey robe" are true, while "Gandalf the Grey is a short, mean shopkeeper living in London" are false. This semantic behavior becomes intelligible once we allow that objects can have properties and be the subject of true or false propositions without requiring existential commitment.

Meinong's classical Theory of Objects has been the subject of sustained criticism since the early 20th century, beginning notably with Bertrand Russell's seminal objections. Nevertheless, in contemporary analytic metaphysics, there has been a resurgence of interest in Meinongian ideas, particularly in the development of neo-Meinongian frameworks. Among these, Modal Meinongianism stands out as one that incorporates the apparatus of possible worlds semantics to address several of the limitations associated with the original theory.

The central proposal is to interpret formulas of the form $\mathcal{R}_a p$ as expressing that the proposition p is realized at the possible (or fictional) world a. This interpretation allows for a nuanced treatment of predication and quantification involving non-existent objects. Additionally, I propose a preliminary definition of 'being' using the existential quantifier as a primitive notion. In our formulation, being is defined as:

$$B(x) = \exists_a(\mathcal{R}_a(\exists_y(y = x))$$

which states that an object x has being if there exists a world a such that x is realized at a. Notably, this draft will not attempt to provide a solution for the treatment of impossible objects; however, it will indicate a potential direction for future investigation in this area.

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Classical and Native Modalizations of Many-Valued Logics with Non-Deterministic Semantics

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Tags: many-valued logic, modal logic, non-classical logic, paraconsistent logic, semantics of modal logic.

We present a systematic method for constructing modal extensions of many-valued logics using non-deterministic semantics. Our approach is illustrated with two case studies: the trivalent Łukasiewicz logic (**Ł3**) [10] and Dialethic Belnap-Dunn logic (**dBD**) [11]. Building on earlier work on non-deterministic matrices (Nmatrices) [1, 3, 7, 6, 13, 14], we show how to modularly add modal operators to these logics while preserving desirable meta-theoretical properties.

Two distinct strategies of modalization are developed. The *classical modalization* introduces a binary modal dimension to the original logic, pairing each truth value with a modal status of either necessary truth or necessary falsity. This yields a two-dimensional semantics that retains the behaviour of the base logic while adding classical modal reasoning. In contrast, the *native modalization* encodes modal status directly within the many-valued structure of the base logic itself, using its original truth values to represent modal notions. This strategy refines earlier many-valued modal approaches such as those of Fitting [9, 8].

We provide full soundness and completeness proofs for the resulting modal systems, adapting standard canonical model constructions to the non-deterministic setting. In particular, we prove that both modalizations of **Ł3** and **dBD** are sound and complete with respect to their intended Nmatrix semantics. For the native modalizations, we further propose specific axiomatizations that guarantee a faithful interaction between modal operators and the indeterminacy inherent in the underlying many-valued logic.

Our work contributes to the broader programme of extending modal logic techniques to non-classical logics, and highlights the flexibility of Nmatrices in capturing a wide variety of modal behaviours. In addition to clarifying the modal potential of Łukasiewicz and Belnap-Dunn logics, the framework we develop can be applied to other many-valued systems, including connexive [2, 15] and paraconsistent logics [4, 5, 12].

Our results suggest that non-deterministic modalization offers a promising route toward unifying modal reasoning with the rich landscape of many-valued and non-classical logics. Future work will explore extending this approach to quantified settings and to systems combining modality with other non-classical features.

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From Second-Order Quantification to Second-Order Definite Descriptions

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Tags: philosophical logic, proof theory.

First-order quantifiers range over objects, while second-order quantifiers range over properties of objects or relations between them. Building on this distinction, we introduce the notion of second-order definite descriptions—a generalization of first-order definite descriptions, which denote unique objects satisfying certain properties. Traditionally, definite descriptions are formalized in first-order logic using terms of the form $\iota x \varphi$, where ι is a description operator denoting the unique x such that the formula φ holds. In this paper, we propose a second-order extension of this concept: expressions of the form $\iota X \varphi$, where X ranges over properties or relations. The aim is to investigate how meaningful statements about "unique relations" can be expressed within second-order logic while preserving key ideas from Russell's theory of definite descriptions [2]. Although full second-order logic is incomplete, its Henkin-style fragment admits completeness. We develop our approach within this fragment and provide a cut-free sequent calculus formalization of the resulting system based on the sequent calculus from [1]. We also briefly discuss how second-order variants of other theories of definite descriptions could be formulated.

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A Semantics of the Basic Modal Language Based on a Generalized Rough Set Model

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Tags: modal logic, non-classical logic, semantics of modal logic.

Modal logic plays a crucial role in the formal reasoning of rough set models. The use of modality operators provides an abstract characterization of approximation operators based on approximation spaces. The *Subset Approximation Structure* (SAS) generalizes the notion of an approximation space by enabling reasoning over selected subsets of the universe. In [1], we introduce a generalization of SAS, which we call an *approximation frame*. This frame consists of a universe W, a neighborhood function ρ, and a binary relation R, in contrast to SAS, which uses a collection of non-empty subsets σ in place of ρ.

We propose a novel semantics for the basic modal language by interpreting the necessity operator $\Box$ as the *necessity lower approximation operator* defined over the approximation frame (W, ρ, R). Within this framework, we construct sound and complete axiomatic systems for several important classes of approximation frames and investigate key model-theoretic properties such as *bisimulation*, *invariance*, and *modal definability*. To establish completeness, we employ a modified step-by-step proof [2] method tailored to our framework. Furthermore, this study leads to the formulation of the necessity lower approximation operator on SAS by identifying a correspondence between SAS and the standard approximation frame.

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From Relevance Logic to Projective Geometry

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Tags: relevance logic, application of logic, philosophical logic.

The relationship between relevance logic and projective geometry is both profound and indisputable.

The aim of this paper will be to show that starting only from the Veblen-Young axiom we can arrive at geometry.

The relevance logic RK arises through the extension of the system R by the addition of the Duns Scotus' law. Importantly, the incorporation of this principle as an additional axiom does not result in a collapse into classical logic, a fact established by Robert Meyer. Within the framework of projective geometry, one encounters rich algebraic structures—most notably projective planes—which facilitate the construction of models for relevance logics in a manner that faithfully preserves their intended semantics.

This talk aims to elucidate the deep structural connections between projective planes and the ternary relation R. It will be demonstrated how projective planes can serve as intuitive and geometrically grounded visualizations of abstract logical structures. Consequently, several significant interrelations between these mathematical constructs will be explored and clarified.

Intermediate Logics Comparison

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Tags: intuitionistic logic, intermediate logics, semantics of intuitionistic logic.

Tabular intermediate logics form a class of intermediate logics defined by finite posets regarded as Kripke frames [1]. For a poset $\mathbb{P}$, let $L(\mathbb{P})$ denote the associated logic. We study the decision problem $\mathsf{LogContain}$: given finite posets $\mathbb{P}$ and $\mathbb{Q}$, determine whether $L(\mathbb{P}) \subseteq L(\mathbb{Q})$.

By a classical result due to Jankov and de Jongh, this problem is closely related to $\mathsf{SPMorph}$: the existence of a surjective p-morphism from $\mathbb{P}$ to $\mathbb{Q}$. Both problems belong to the complexity class NP.

We establish a certain connection between graph homomorphisms and p-morphisms. Specifically, we define a transformation that assigns to each graph $\mathbb{G}$ a poset $\mathsf{Pos}(\mathbb{G})$ in such a way that a surjective locally surjective homomorphism from $\mathbb{G}$ to $\mathbb{H}$ exists if and only if there is a surjective p-morphism from $\mathsf{Pos}(\mathbb{G})$ to $\mathsf{Pos}(\mathbb{H})$. This correspondence allows us to derive NP-completeness of several restricted cases of $\mathsf{LogContain}$ and $\mathsf{SPMorph}$. Notably, we exhibit an 18-element poset $\mathbb{Q}$ for which deciding whether $L(\mathbb{P}) \subseteq L(\mathbb{Q})$ is NP-complete.

We also identify a tractable case: we design a polynomial-time algorithm solving $\mathsf{LogContain}$ and $\mathsf{SPMorph}$ when the first poset in the input is a tree.

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Two New Explosive Logics and Their Transformation to Paraconsistent Logics

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Tags: algebraic semantics, non-classical logic, paraconsistent logic, proof theory.

Contrary to classical logic, common-sense reasoning does not endorse triviality in presence of contradictory information - that is, information which contains both a statement and its negation.

By a logic $\mathcal{L}$ we mean a tuple $(FOR, \vdash)$, where FOR represents the set of all formulae over an alphabet containing the connective $\neg$ for negation, and $\vdash$ represents the consequence relation on FOR; that is, $\vdash \subseteq \mathcal{P}(FOR) \times FOR$, where $\mathcal{P}(FOR)$ denotes the power set of FOR. The principle of explosion, usually known as Ex Contradictione Quodlibet (ECQ), is given as follows:

$$\Gamma \cup \{\alpha, \neg\alpha\} \vdash \beta, \text{ for all } \Gamma \cup \{\alpha, \beta\} \subseteq FOR.$$

In [12], we have surveyed some major approaches following which paraconsistent systems are developed. In this study, we focus on one of the approaches, known as *transformation approach*. Under this approach, four techniques are discussed, namely, *paraconsistentization* [3], *left variable inclusion companion* [2], *restricted rules companion* [1] and *plurivalence* [11]; using any of these approaches one can transform an explosive logic $\mathcal{L}$ to a non-explosive one. Here, we shall construct two new non-classical explosive logics and transform them into paraconsistent systems using paraconsistentization and left variable inclusion companion techniques.

Based on the study in [4], below a list of combinations leading to explosion are summarized on different fragments of a propositional language. The conditions considered in the combinations are presented following sequent calculus of classical propositional logic (CPL) along with the axiomatic sequent $(Ax0) : \Gamma, \alpha \Rightarrow \Delta, \alpha$. It is observed that some known explosive logics such as CPL, intuitionistic propositional logic (Int), Łukasiewicz's three-valued logic ($Ł_3$) [8], Kleene's three-valued logic (K_3) [7], Gödel's three-valued logic (G_3) [5], Post's three-valued logic (P_3) [10], Bochvar's three-valued logic (B_3) [6], non-truth functional logics $< 34[2] >$ and $< 4[2] >$ by Roman Tuziak [13] can be placed under some of the combinations.

$(Comb_1)$ $\neg$-L
(Examples: $CPL,\ K_3,\ G_3, Ł_3,\ B_3,\ P_3,\ Int,\ < 34[2] >, < 4[2] >$).

$(Comb_2)$ $\neg$-R + $\neg$-L + cut
(Examples: $CPL,\ G_3$).

($Comb_3$) $\neg\neg$-E + Subminimality
(Example: CPL).

($Comb_4$) $\vee$-R + cut + (DS)
(Examples: $CPL, K_3, G_3, L_3, B_3, P_3, Int, < 34[2] >, < 4[2] >$).

($Comb_5$) $\neg$-L + $\vee$-R + $\vee$-L + cut
(Examples: $CPL, K_3, G_3, L_3, B_3, P_3, Int, < 34[2] >, < 4[2] >$).

($Comb_6$) $\wedge$-explosion + $\wedge$-R + cut
(Examples: $CPL, K_3, G_3, L_3, B_3, P_3, Int, < 34[2] >, < 4[2] >$).

($Comb_7$) $\alpha \wedge \beta \Rightarrow \neg(\neg\alpha \vee \neg\beta)$ + Subminimality + $\neg\neg$-E + LEM + $\wedge$-R + $W \Rightarrow$ + cut
(Examples: CPL).

($Comb_8$) $\vee$-R + $\rightarrow$-L + $\neg\alpha \vee \beta \Rightarrow \alpha \rightarrow \beta$ + cut
(Examples: CPL, K_3, Int) .

($Comb_9$) $\neg\neg$-I + $\rightarrow$-L + $\alpha \vee \beta \Rightarrow \neg\alpha \rightarrow \beta$ + $\vee$-R + cut
(Examples: $CPL, K_3, G_3, L_3, Int, < 34[2] >, < 4[2] >$).

Among the above mentioned rules $W \Rightarrow$, cut, $\neg$-R, $\neg$-L, $\vee$-R, $\vee$-L, $\wedge$-R, $\wedge$-L, $\rightarrow$-R, $\rightarrow$-L are considered in their respective forms as given in CPL [9] and the rest are as follows.

($\neg\neg$-I) : $\alpha \Rightarrow \neg\neg\alpha$

($\neg\neg$-E) : $\neg\neg\alpha \Rightarrow \alpha$

(DS) : $\alpha, \neg\alpha \vee \beta \Rightarrow \beta$

($\wedge$-explosion) : $\alpha \wedge \neg\alpha \Rightarrow \beta$

(LEM) : $\Rightarrow \alpha \vee \neg\alpha$

(Subminimality) : $\frac{\Gamma, \alpha \Rightarrow \beta}{\Gamma, \neg\beta \Rightarrow \neg\alpha}$.

In this presentation, two new non-classical explosive logics corresponding to $Comb_6$ and $Comb_8$ are proposed. The logics are formulated following sequent calculus style proof theory and the soundness-completeness results are proved with respect to an algebraic semantics. Two new classes of algebras are defined for this purpose. Their properties are studied, and examples are given. It is found that these algebras are neither Heyting algebra, nor quasi-Boolean algebra, nor Boolean algebra.

Finally, these logics are transformed into paraconsistent logics using paraconsistentization and left variable inclusion companion techniques. Some properties of these are checked.

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The Dispute Between the Lvov-Warsaw and Kraków Schools in the Interwar Period: On the Nature of Mathematical Knowledge

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Tags: history of Polish logic, logic and artificial intelligence, philosophical logic.

Philosophical logic has developed through various intellectual traditions. One of the most significant among these is the Polish school, which contributed greatly to early 20th-century developments in logic, set theory, and semantics [10]. As Jan Woleński [12, 11] has documented, the Polish tradition – represented by figures such as Jan Łukasiewicz, Stanisław Leśniewski, Alfred Tarski, and others – was instrumental in laying the foundations for many of the formal and philosophical advances in logic that would influence both European and Anglo-American traditions.

Meanwhile, we can say that logic is a form of integral knowledge—integral both historically and conceptually, as emphasized by Lemanski [6, 5]. Its integrality signifies its capacity to unify diverse domains of reasoning under general principles and formal systems. This is what makes logic not only foundational to all rational inquiry but also uniquely suited to serve as a meta-theoretical framework.

This perspective on logic as an integrated and overarching discipline underlies Jean-Yves Beziau's project of *universal logic*. According to Beziau [1], universal logic is not a specific new logical system, but rather a general theory of logics—a formal study that aims to analyze and compare all logical systems in terms of their mathematical structure [2, 3]. It is an ambitious attempt to treat the diversity of logical systems—classical, non-classical, modal, paraconsistent, fuzzy, relevant, and others—under a single theoretical umbrella.

In a broader definition, Beziau describes logic as a multifaceted domain encompassing both technical and philosophical dimensions:

> Logic as a field is the study of the various systems of logic; their applications, interpretations, and meanings; and the historical and philosophical aspects of the theory of reasoning, including the art of thinking, argumentation, fallacies, and paradoxes [4, p. 151].

This definition underscores that logic is not just a formal enterprise, but also deeply involved with contextual meaning, applied reasoning, and philosophical reflection. It reflects a shift from viewing logic as a fixed monolithic system (such as classical logic) to recognizing it as a dynamic landscape of interrelated systems with different purposes and domains of application.

As a possible path forward in the development of universal logic, we propose considering fragments of logical systems—so-called *logemes*—as simplicial complexes (a kind of topological spaces) [8, 9]. By isolating small structural units of logic and treating them as objects within a topological framework, we can analyze how local logical properties interact and how systems relate to one another globally. This topological interpretation enables a unified comparative analysis of various logical systems – such as classical, intuitionistic, and many-valued logics—even when their syntax, semantics, or inference rules differ significantly. Such an approach offers a universal language for understanding and translating between logical fragments, further reinforcing the vision of logic as a unified yet pluralistic field.

This topological or structural perspective aligns with the foundational spirit of universal logic, as it seeks common ground between logical systems without reducing them to a single paradigm. Instead of asking which logic is 'correct,' it asks how different logemes (fragments of different logics) relate, interact, and cohere within topological spaces. This approach, based on topology, can be called a *phenomenological approach to philosophical logic*. It places emphasis not merely on the whole formal structures of logics themselves, but on how these structures are experienced, constructed, and interpreted as meaningful fragments (logemes). In this sense, logics are not treated as abstract, detached calculi of symbols, but as something that emerges within a world of meaning, shaped by conceptual frameworks, linguistic intuitions, and evolving mathematical practices.

From a phenomenological perspective, logical systems are not imposed from above or *a priori* but arise from the lived experience of reasoning, the way we conceptualize relations, identity, quantity, and inference. They are rooted in intentional acts—the conscious orientation toward logical forms and their applications within models, discourses, and problems. In this way, logic becomes an expression of how thinking gives form to itself through structured, repeatable operations.

This approach echoes the views of *Stanisław Zaremba* (1863–1942), a Polish logician who emphasized that mathematical propositions only make sense in context and who challenged the idea of content-free logical propositions. The phenomenological approach to logic thus challenges strict logical formalism, advocating instead a view in which logic is localized and intentional. In this light, the phenomenological approach supports a flexible, pluralistic conception of logic as a unified knowledge, where different logical systems reflect different ways of perceiving and structuring reality, which can be analyzed as logemes in different topological spaces. Logical truths are not simply formal necessities, but are grounded in how we, as thinkers, engage with structures of meaning—whether in mathematics or philosophy. In everyday life, we engage not with entire logical systems, but rather with isolated logemes—fragments or elements of logic that appear in specific contexts. These may include simple inferences, assumptions, or conditional statements, rather than the structured, comprehensive logic studied in formal systems.

The *foundations of mathematics* began to take shape in the early 20th century, driven by the aspiration to establish a single, unified formal logic capable of underpinning all mathematical reasoning. This quest for a comprehensive logical framework reached a particularly critical and formative moment in Poland, where two major intellectual traditions—the Kraków School and the Lvov-Warsaw School—engaged in a profound philosophical and logical confrontation [13]. At the heart of this debate stood two influential figures with opposing views: Stanisław Zaremba, a mathematician firmly grounded in the classical tradition, who maintained that a single logic for all of mathematics was neither possible nor desirable; and Jan Łukasiewicz, a logician and philosopher renowned for his groundbreaking contributions to symbolic logic, who argued that a unified logical system was not only possible but essential, drawing inspiration from the logicist program of Frege and Russell.

Hence, this debate was not merely about technical definitions or symbolic formulations in logic. It concerned the very nature of logic in mathematical knowledge: *Is mathematics, at its core, a system of logical truths constructed from atomic propositions linked by logical connectives?* Can all of mathematics be reduced to logic by adding non-logical axioms, as the proponents of logical atomism believed? Or must mathematics be understood within the context of specific structures and models, as Zaremba argued, anticipating ideas that would only later be formalized in model theory?

The discussion drew in other leading thinkers, including Kazimierz Kuratowski, Tadeusz Czeżowski, and even caught the attention of many French and Soviet mathematicians, such as Nikolai Luzin, one of the founders of descriptive set theory in the USSR.

Jan Łukasiewicz believed in the existence of a single, unified logic capable of grounding all mathematical reasoning. Stanisław Zaremba, however, challenged this view in 1917, see [15], arguing that such a universal logic was not possible because mathematical discourse includes not only true and false propositions, but also *contentless propositions*—statements that lack any semantic content. According to Zaremba, these contentless propositions have no corresponding models, and as a result, their negation does not affect their truth value.

Interestingly, although Łukasiewicz would go on to propose a formal system of three-valued logic in 1920, see [7]—without referencing Zaremba—it was in fact Zaremba who first introduced the idea of a three-valued logic in 1917 specifically for mathematical reasoning during his debate with Łukasiewicz. Thus, while Łukasiewicz is often credited with the development of multi-valued logic, the conceptual roots of this approach can be traced back to Zaremba's earlier critique of logical monism in mathematics.

Although Zaremba was largely dismissed by his contemporaries and the academic circle around him eventually disintegrated, his ideas have proven to be remarkably forward-looking. At the time, the prevailing philosophical and mathematical climate—especially under the influence of the logicist program led by Frege, Russell, and later embraced by Łukasiewicz—favored the pursuit of a single, unified logical foundation for all of mathematics. In contrast, Zaremba's skepticism was seen as out of step with the dominant intellectual trends.

Yet with the benefit of hindsight, many of Zaremba's insights now appear prophetic. Today, it is widely accepted among philosophers of mathematics and logicians that there is no single, all-encompassing logic that can serve as the universal foundation of mathematics. Classical logic, like any other, is not capable of fulfilling this role. The dream of reducing mathematics entirely to logic has proven untenable, as different areas of mathematics often rely on different logical frameworks—classical, intuitionistic, modal, and beyond—depending on context and foundational assumptions.

But Zaremba's intuition went even further. He suggested not merely that multiple logics might be needed, but that in practice, we do not reason using entire formal systems at all. Instead, we operate with fragments of reasoning, or logemes—discrete, context-bound units of logical thought. This idea anticipates contemporary views in both cognitive science and the philosophy of logic, where the emphasis has shifted toward understanding how logic functions in situated reasoning, communication, and problem-solving, rather than as an abstract, monolithic system.

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Arguments and Syntactic Categories

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Tags: philosophy of logic, logical semiotics, argumentation theory.

According to Ch. S. Peirce's classification of signs [2], there are *a priori* three types of signs determined by the mode of reference of the so-called interpretant to the so-called representamen: rhemes, dicisigns and arguments. However, the theory of syntactic categories [1] provides only two basic, primary categories: names and sentences (the other categories, i.e., functors and operators, are secondary to them). Names are rhemes, sentences are dicisigns, however to arguments no separate basic category corresponds. In this paper I propose to fill this gap in syntactic categories, which is revealed by the semiotic perspective of Peirce's concept of sign, and to extend the theory with an additional basic category, namely arguments.

Among the reasons behind such an extension, it is worth emphasizing those that directly inspire the present consideration. They have been provided by a study of argument syntax [3, 4]. The study concerns various operations on arguments. On the basis of this research some functors will be indicated that correspond to such operations. Thanks to these functors, linguistic expresions can still be maintained to be finite strings of primitive signs, preserving classical, linear concept of language. Besides, other reasons for the theoretical usefulness of the extension in question will be discussed.

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On Tarski's Theory of Temporally Extended Objects

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Tags: formal ontology, philosophy of science, temporal logic.

In his 1937 book *The Axiomatic Method in Biology* (see [4]), J. H. Woodger proposed a formal axiomatic system as a framework for biology. Among the primitive concepts adopted by Woodger are the relational notion of "being a part of", the relational notion of "preceding in time" and the concept of an "organized unit". These notions were analyzed by Alfred Tarski in *Appendix E* (see [3]), included in Woodger's book. There, Tarski described a formal system based on classical mereology, in which he examined the consequences of assigning different properties to the notions of preceding in time and organized unit than those originally formulated by Woodger.

The aim of the presentation is to introduce the extended mereological system described by Tarski in the appendix to J. H. Woodger's book and to formulate this system in a contemporary formal language. A relatively simple model for the theory will be proposed. Additionally, the paper will address the question of whether Tarski's theory can be interpreted as a version of the ontological view known as four-dimensionalism (see [2]).

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Invertible Syntactic Reductions in Transitive Modal Logics

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Tags: modal logic, non-classical logic.

A *reduction* for a sequent α is a finite sequence of finite sets of sequents

$$\Phi_1 = \{\alpha\}, ..., \Phi_z$$

such that for every $1 \leq i < z$, Φ_i is valid (that is, each of its sequents is) iff so is its immediate successor, and Φ_z consists of some simple sequents. Such reductions are *cycle-free*. We remark that our reductions resemble modal Socratic proofs (studied in [1]), but modal Socratic proofs involve semantic labels, whereas our reductions are *label-free*.

By using the result on the modal logic **S4F** (see [2]), we illustrate how our method works for Mints-style normal forms. (Mints-style normal forms were developed to Modal Logic by Mints from some techniques introduced by Wajsberg in Intuitionistic Logic [5].) For every formula ϕ, a Mints form α can be constructed with the property that ϕ is valid iff α is. (A sequent $\Phi \vdash \Psi$ is valid iff so is its formula $\bigwedge \Phi \rightarrow \bigvee \Psi$.)

In our reduction procedures, we start with initial forms, which are simple modifications of Mints-style normal forms. By an *initial form* we mean a sequent

$$\alpha = \Box\Theta; \Box\Delta; \Box\Gamma \vdash a$$

where $\Theta = \{\Box a \leftrightarrow a'\}$ and a, a' are atoms (that is, propositional variables or the constant $\bot$ (the false)), Δ is a finite (possibly empty) set of formulas of the kind $\Box b \leftrightarrow b'$ with atoms b, b', and Γ is a finite (possibly empty) set of $\Box$-free formulas. We put: $\mathsf{M} = \{b : \Box b \leftrightarrow b' \in \Delta\}$ and $\mathsf{M}' = \{b' : \Box b \leftrightarrow b' \in \Delta\}$ and $\mathsf{M}^a = \{\Box b \rightarrow a : b \in \mathsf{M}\} \cup \{\Box a \rightarrow b : b \in \mathsf{M}\}$.

A *reduction system* consists of reduction rules and simple sequents. A *reduction rule* $\frac{\Phi}{\Psi}$ is sound iff we have: Φ is valid iff Ψ is. (So, in a reduction rule, the sets Φ, Ψ are, in fact, meta-level conjunctions.) We say that a reduction system is *sound* iff so is each of its reduction rules, and it is *complete* iff every initial form has a reduction.

By a *final form* we mean a sequent

$$\omega = \Box\neg\Box\mathsf{A}; \Box\Gamma \vdash \bot$$

where A is a finite nonempty set of atoms and Γ is a finite set of $\Box$-free formulas.

Our final forms have the nice property that:

ω is valid iff so is every $\Box$-free sequent $\Gamma \vdash c$ $(c \in \mathsf{A})$.

By a *normal form* we mean a sequent

$$\psi = \Box(\neg\Box\mathsf{A}; \mathsf{M}^a; \Theta); \Box\Delta; \Box\Gamma \vdash a; \Box\neg\Box a$$

where A is a finite (possibly empty) set of atoms.

By a *special normal form* we mean a normal form with the property that every $\Box$-free sequent $\gamma_c = \Gamma \vdash c; a'; \mathsf{M}'$ is nonvalid ($c \in \{a\} \cup \mathsf{M}$).

We have: *If a normal form ψ is not special (that is, some γ_c is valid), then ψ is valid.*

Our reduction system for **S4F** is the following.

- *Simple Sequents*: Every final form.

- *Reduction Rules*:

  Normalization Rule: $\frac{\{\alpha\}}{NF(\alpha)}$, where $NF(\alpha)$ is the set of normal forms and final forms determined by an initial form α (see [2]).

  Elimination Rule: $\frac{NF(\alpha)}{SNF(\alpha)}$, where $SNF(\alpha)$ results from $NF(\alpha)$ by deleting each normal form that is not special.

  Final Rule: $\frac{SNF(\alpha)}{FIN(\alpha)}$, where $FIN(\alpha)$ results from $SNF(\alpha)$ by replacing every special normal form ψ in $SNF(\alpha)$ with the final form determined by ψ.

More results of this kind can be found in [3], [4].

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Decidability of Pairing Function Theory

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Tags: algebraic logic logic and the foundations of computer science.

Based on previous research on the decidability of Grzegorczyk's concatenation theory, which focused on the decidability of concatenation theory without the axiom of associativity, the question is raised about the necessary and sufficient conditions for a pairing function theory to be essentially undecidable. As is well known, certain theories, such as Cantor's pairing function theory, are decidable (see [1]). Others, however, are not (see [2]). In the presentation, the question is formulated in a general way. The connection between the theory of pairing functions and associativity and commutativity is pointed out. Some previous attempts to formulate necessary and sufficient conditions for pairing functions are shown, as well as some remarks about unordered pairing functions.

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Logic of Algorithmic Knowledge

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Tags: epistemic logic, logic and artificial intelligence, temporal logic.

In the study of knowledge representation within logical frameworks, it is standardly assumed that agents enjoy logical omniscience: they are taken to possess immediate access both to all tautologies and to all logical consequences of their epistemic state. Such an assumption is acceptable insofar as one seeks to characterize idealized agents. Under this idealization, no account is given of the computational resources or temporal effort required for the derivation of consequences. In contrast, when the aim is to model the epistemic states of real cognitive agents, one must take into account not only the knowledge an agent explicitly possesses at a given moment, but also the set of conclusions the agent is capable of deriving under specified resource constraints and within bounded time. In our presentation we shall consider a framework of temporal algorithmic logic, in which the temporal dimension required for the derivation of consequences from an agent's current epistemic base is explicitly incorporated. The language of this system permits the formal modeling of epistemic processes of resource-bounded agents. Such a goal cannot be attained without acknowledging the finitude of epistemic resources, for actual agents do not have access to unbounded memory or unbounded computational power. Standard epistemic modal logic—widely employed in the formalization of knowledge—is inadequate for this purpose, as it presupposes logical omniscience and thus fails to account for reasoning under resource-boundedness. The system of algorithmic logic under discussion is designed precisely to overcome these limitations and to provide a formal apparatus for the representation of epistemic states subject to computational and temporal constraints.

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An Infinite System for Branching Time Logic, Strongly Complete in the Semantics of Finitely Branching Infinite Trees

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Tags: infinitary logic, philosophical logic, temporal logic.

Applications of temporal logics motivate the search for their systems that satisfy the strong completeness theorem. For logics that are not compact, logicians formulate infinitistic systems in which ω rules of inference are used. The usefulness of such rules in extra-logical discourse is currently being discussed in philosophy (cf. [4]), and the systems with ω rules are studied in formal epistemology and computer science (cf. e.g. [1]).

For the logic of discrete linear time, G. Sundholm [3] was the first to provide such a system, and he used an ω rule according to which if at each moment following n formula A is true, then one can conclude that A is true in an infinite chain of time points starting from n. Sundholm's proof of the strong completeness theorem is based on the fact that the introduced ω rule can be finitized in such a way that it is possible to use Lindenbaum's construction. We build an infinitistic system for branched discrete time logic UB formulated in [2], which is a fragment of CTL. We take three primitive ω rules in which formulas containing operators $\exists\Box$ (*in some course of the future it will be so that ...*) and $\forall\Box$ (*in every course of the future it will be so that ...*) are deduced from infinite sets of premises. We limit the semantics of infinite trees to infinite but finitely branching trees. Like Sundholm, we use the appropriate finitizations of our ω rules. Thanks to restricting our semantics, König's Lemma assures the soundness of our rules and may be used in the construction of the model. In the proof of strong completeness we use two kinds of syntactic inference relations: $\vdash_f$ and $\vdash_\omega$. According to the first one, if a theory forces at a tree level n more than $f(n)$, it is inconsistent. The second one, a theory is inconsistent if it forces infinite branching at some level of a tree.

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Formal Logic and Machine Learning: Strange Bedfellows?

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Tags: automated reasoning and automated theorem proving, logic and artificial intelligence.

Logic is no longer at the centre of AI research: after all, it is the statistical and, not the rule-based, approach to AI that have taken the world by storm. According to people like the Nobel-prize winner G. Hinton, there is no place for symbolic reasoning anymore. However, the more recent advances in the field of LLMs show a way of merging logic and machine learning. The days of logic's undisputed reign might be long gone but we argue that the best path forward is to combine these approaches. I will first show how *supervised learning* can be beneficial in logic research (e.g., to [1]). Then I will talk about how to apply formal logic to LLMs. One of the biggest problems facing such systems is how to reason coherently about user's queries (see [2], [3]). I will share my experiences related to improving the reasoning capabilities of Bielik, one of the first Polish LLMs ([4]). Finally, I will discuss the potential of extending LLMs capabilities with *knowledge graphs* and formal logical reasoning ([5], [6]).

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A Philosophical Examination of Infinitary Rules of Inference

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Tags: infinitary logic, philosophy of logic.

An infinitary inference rule is a rule with infinitely many premises. One well-known example is the ω-rule, which allows us to infer $\forall x\, A(x)$ from the infinite set of premises about natural numbers $A(0), A(1), A(2), \ldots$. The rule was introduced by Hilbert in the early 1930s, though a closely related form was proposed slightly earlier by Tarski [5].

Such rules play an important role in arithmetic and in logic. However their applications raise the following question: How can finite agents use rules with infinitely many premises? Carnap defended the use of the infinitary rules by appealing to his principle of tolerance, though some critics, including Beth and Church, raised concerns [2, 4].

In recent philosophical discussions, there has been renewed interest in whether infinitary rules of inference can be legitimately employed by finite agents. The question is linked with a logical inferentialism and a mathematical conventionalism. Inferentialists argue that the meanings of logical constants are determined by their inferential roles, and some, like Brîncuş, suggest that our use of quantifiers like 'for all' implicitly commits us to infinitary reasoning patterns that extend beyond finite proof systems [2]. Similarly, conventionalists following Carnap claim that the adoption of infinitary rules can be justified within a linguistic framework, provided they fulfill expressive and inferential purposes [1, 4].

However, critics argue that such rules exceed human cognitive capacities unless one accepts the controversial view that humans possess uncomputable reasoning abilities [1, 3, 6]. In my presentation, I argue that, under certain assumptions, the use of infinitary inference rules may be philosophically justified and may even turn out to be necessary for capturing aspects of logical reasoning that cannot be adequately expressed within purely finitary systems.

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A Truthmaker Semantics for Mixing Classical and Quantum Connectives

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Tags: non-classical logic, philosophical logic, quantum logic, substructural logic, truthmaker semantics.

In this talk we present a way in which it is possible to make sense of hypertintensional talk about both orthodox quantum and classical phenomena at the same time. Since Von Neumann and Birkhoff's seminal work formalizing quantum mechanics (see [1]), it is well-known that, given some natural ways in which one can define disjunction and conjunction, observable quantum phenomena are not structured according to a boolean algebra, as classical propositions are. More particularly, the distributivity of disjunction and conjunction does not hold in lattices of quantum propositions. A quantum proposition is a statement stating that a quantum system is in a state such that an observable quantity of the system has a specific value v at that quantum state (i.e. such that it is in an eigenstate of the Hermitian operator corresponding to that quantity with eigenvalue v).

We devise an *exact* truthmaker semantics. By *exact* we mean that sentences are verified and falsified by those states[8] that .are entirely responsible for the verification or the falsification, not just some part of it. The sentence is then not only merely true or false at such verifying or falsifying states, but it is false or true in virtue of all of the parts of these states. As it has become standard since Kit Fine's work on truthmaker semantics (see for example [2]), we evaluate sentences at states in a mereological state space $\langle S, \sqsubseteq \rangle$, where S is the set of all states and $\sqsubseteq$ the partial order relation specifying whether states are a (possibly improper) part of each other. The states are the objects that can make sentences true or false (the truth-makers and falsity-makers). They can contain enough information to specify the configuration of an entire world/system (like worlds in possible world semantics), but can also contain just a part of that (i.e. partial information) and even inconsistent information. By means of the parthood relation we can define a *fusion* operation $\sqcup$ by considering the lowest upper bound of two states (the smallest state including all the fused states).

The easiest way to go from ordinary possible-worlds or algebraic semantics to exact semantics is by taking the truthmaker states to be sets of ordinary propositions (an ordinary proposition is a set of possible worlds or an object in an appropriate algebra). In the quantum context, it makes sense to take a state to be a set of linear subspaces of Hilbert space. 'Particle 1 is spin-up in the x direction and particle 2 is spin-down in the y direction' is made true by a state corresponding to a set that contains two subspaces (the vectors representing particle 1 x-spin-up and the vectors representing particle 2 y-spin-down). If

[8]From here on, the word *state* no longer refers to quantum states in the ordinary sense, and it is only used in the specific way in which the word is used in the truthmaker semantics literature, as a container of information.

we add yet another vector space to that set of subspaces, thus adding information to it, the state does not necessarily yield an exact truthmaker for that sentence. Other representations are perfectly permissible, as long as states can capture enough information to support the truth or falsity of statements.

Slightly differing from Fine's work, we modalize the state space not by adding a single set of passible states to the state space (as Fine does), but by adding two sets of states, the exactly self-orthogonal states *Ort* and the exactly impossible states *Imp*, such that $Ort \subseteq Imp \subseteq S$, resulting in a quadruple $\langle S, \sqsubseteq, Ort, Imp \rangle$ as the modalized state space. We call two states *incompatible* if their fusion includes an exactly impossible state and we call two states *orthogonal* if their fusion includes an exactly self-orthogonal state. A state s *Imp*-excludes a set of states T if the fusion of s with any member of T is in *Imp*. *Ort*-exclusion is defined similarly.

Incompatibility means and functions exactly as in Fine's work. States being orthogonal should be interpreted as being, not merely factually, but also *noticeably* incompatible. In quantum contexts, this means that states are not just different in Hilbert space but that a single measurement can distinguish between the states, in a fully classical context orthogonality could be seen as identical to incompatibility. In cases of vague predicates P, we could consider that, for borderline cases b, Pb and not-Pb are incompatible, but not orthogonal, because we cannot reliably make the difference between the two. In cases of indeterminacy (such as future-contingency cases) we could say that 'it rains tomorrow' and 'it does not rain tomorrow' are incompatible, but not orthogonal. In general, orthogonal states are different and distinguishably so, but, while a system cannot be in two non-orthogonally incompatible states s_1 and s_2 at the same time, there is no way of reliably noticing the difference between the two states (with respect to some criterion for reliable notification).

A truthmaker model in our sense fixes a modalized state space and a set of verifiers and falsifiers for all primitive sentences. Verification and falsification is then extended to all sentences by the same clauses as those given by Fine (e.g. a disjunction is verified by the verifiers of any of its disjuncts and falsified by the fusion of the falsifiers of the disjuncts).

The most interesting differences between quantum and classical cases become visible when we consider our notions of *hyperpropositions* (i.e. hyperintensional propositions) and *distinguished hyperpropositions* (i.e. hyperintensional propositions only referring to noticable differences). The first are defined by the closure of sets of states under double *Imp*-exclusion and the second by closure under double *Ort*-exclusion. If a set of states are the ordinary verifiers/falsifiers of a formula, then the smallest hyperproposition including that set are the indirect verifers/falsifiers. The upshot of taking into account these two kinds of hyperpropositions is that they can make finer distinctions than traditional classical or quantum logic: $r \wedge (p \vee \neg p)$ and $r \wedge (q \vee \neg q)$ have a different set of states as indirect truthmakers (this is the main advantage of choosing truthmaker semantics rather than traditional semantics), but one can also coarse-grain the structure of hyperpropositions readily into a boolean algebra and the structure of distinguished hyperpropositions into traditional orthomodular lattices. This extends a result proven in my [3].

In the talk I will define the semantics, explain its basic feature, demonstrate the links with existing systems of classical and quantum logic. I will also show how this semantics gives a subtle treatment of the validity of excluded middle in quantum logic (and in classical logic truthmaking—consider that a state that contains no information about roundness will not directly make '*a* is round' either true or false). Then I will argue that this mixed approach avoids having to completely reject classical logic when taking orthodox quantum reasoning seriously. Finally I will suggest ways in which this treatment can teach us something about ways of dealing with indeterminacy in general (beyond the quantum). If time permits, I will also add a few words on the prospect and benefits of adding modalities to the language that express the unitary evolution (by means of quantum gates for example) or measurement projections of quantum systems.

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The Status of Conditional Excluded Middle in Contemporary Conditional Logic

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Tags: conditional logic, counterfactuals, philosophical logic, possible worlds semantics.

After the publication of Lewis's *Counterfactuals* [3] in 1973, it became widely accepted among scholars working on conditionals that Conditional Excluded Middle (CEM)—a central thesis of Stalnaker's logic—cannot be maintained. In recent years, however, several prominent authors have defended the plausibility of this principle (see, for example, [1, 2, 5]). In my talk, I discuss how CEM stands today, nearly fifty years after the famous debate between Lewis and Stalnaker (see especially [4]).

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The Logical Creativity in the Lvov-Warsaw School and Its Reverberation Throughout the World

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Tags: history of logic, history of Polish logic.

Contemporary world literature dealing with logic and its history very often features the term 'Polish logic'. This term was coined by S. McCall [1] to underline the great contribution of Polish logicians of the interwar period, in particular members of the Warsaw School of Logic (WSL) to the development of world logic. The WSL was the famous branch of the Lvov-Warsaw School that laid the foundations of formal logic developed later in the WSL and in certain domains of which Polish scientists played a pioneering role, outstanding in the global scale. A lot has already been written on the main pillars, ideas and significance of the WSL (current items are, e.g., J. Woleński [2], U. Wybraniec-Skardowska [3]). The aim of the presentation is to provide an overview and synthetic discussion of the creative contribution of the logicians of the Lvov-Warsaw School, which led to the development of Polish and world logic in the 20th century and which remains a living link in its influence and functioning in the international arena to this day.

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Questions as Sentential Quantifiers

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Tags: logic of questions, non-classical logic.

1. Introduction

Recent research on declarative and interrogative sentences, done basically in the so called *inquisitive semantics* (see [1] and [2]) aims at an unified and integrated semantic analysis of these two types of sentences. In this talk I propose another way to unify the semantics and interrogatives: both declaratives and interrogatives denote sets of *sentential predicates* (that is sets of sets of sentences) but predicates they denote have different semantic properties.

In this approach the reference to answers (to *yes-no*-questions) is avoided because it seems that there is an infinite number of answers to such questions. For instance the question *Is it true that p* can be answered by any sentence of the form *yes, if q* or *yes, unless q*.

The analogy between declaratives and interrogatives which follows from this approach is similar to the analogy established in the generalised quantifier theory between different types of noun phrases.

2. Sentential categories

Sentential categories are grammatical categories which contain sentences as their part and categories whose syntax and semantics depend on such categories:

(1) That life is sad is a rumour/true/a serious hypothesis.
(2) Whether life is sad is not known/is seriously investigated.
(3) Dan believes/suspects that life is sad.
(4) Dan believes nothing that Bo said/believes except that life is sad.

In (1) we have a sentential noun phrases (SNPs) *that life is sad* and sentential predicate (SPR) *is a rumour/is true/is a serious hypothesis*; in (2) *whether life is sad* is a SNP and *is not known/is seriously investigated* are SPRs and in (3) the SNP is *nothing that Bo said/believes except that life is sad* and the SPR is *Dan believes.*

SNPs form complex sentences (CSSs) with SPRs. Their general syntactic form is indicated by the rule in (5) The rule forming a SNP is indicated in (6) (where + is the concatenation sign):

(5) CSS=SNP+SPR   (6) SNP=COMP+S

The category COMP corresponds in English to items like *that* or *whether*, which are called *complementizers*. Important point is that with the category COMP can be associated a specific meaning since there is clearly a difference between (6a) and (6b):

(6a) Dan believes that (either) life is sad or the earth is flat.
(6b) Dan believes that life is sad or that the earth is flat.

(6a) says that Dan believes that a certain disjunction holds; Dan may have no opinion of which disjunct is in fact true. (6b), it indicates that Dan believes one of the disjuncts.

3. Semantics of sentential categories

The basic ingredient of the model in which sentential categories are interpreted is the set Σ of sentences of a given (interpreted) formal or natural language (see [3]). This set is simplified since its elements contain sentences with maximally one external negation: any sentence of the form $n..nS$, is identified with S or with nS depending on whether there is an even or odd number of n-s.

Sentential predicates are interpreted as subset of Σ: the predicate T (*It is true*) is interpreted by the set of true sentences and the predicate *Dan believes* is interpreted by the set of sentences that Dan believes to be true.

Since SPRs are sets of Boolean objects they have two negations, the Boolean complement and the post-complement which corresponds to the set of their negated elements:

D1: Let P be a sentential predicate. $\neg P$ is defined as $\neg P = \{S : S \notin P\}$. The post-complement of P, $P\neg$, is defined as $P\neg = \{nS : S \in P\} \cup \{S : nS \in P\}$.

The notion of post-complement corresponds to the negation of the sentential argument of a sentence forming operator. It will be used to determine various properties of sentential predicates. For instance the relationship between the predicates K_a and KW_a can be expressed, with the help of the post-complement as $KW_a = K_a \cup K_a\neg$ (where $S \in KW_a$ corresponds to *a knows whether* S).

Post-negation of a SPR can be used to define a class of sentential predicates called *midpoint* predicates [3]:

D2: $P \in SPR$ is a midpoint predicate iff whenever $S \in P$ then $nS \in P$.

Clearly: $P \in MPP$, iff $P = P\neg$.

D3: P is intensional, $P \in INT$, iff $P \neq \emptyset$ and $\forall S \in P \exists S_0 \in P(v(S) = v(S_0)) \wedge S_0 \notin P$, where $v(S)$ is the truth-value of S.

SNPs are interpreted by sentential quantifiers, SQ, that is by sets of SPRs. For instance the SNP *that S* is interpreted by sentential quantifier $THAT(S)$:

D4: $THAT(S) = \{P : P \in SPR \wedge S \in P\}$.

COMPs are interpreted as functions from set of sentences into sets of SQs.

A SPR is consistent iff it whenever it contains S it does not contain nS.

It follows from the above that verbs like *believe, know* forming intensional SPRs denote relations between individuals and sentential quantifiers.

4. Questions

Questions are characterised by a specific set MPPs. The general form of the function denoting an SNPs headed by the complementizer *whether* is given D5:

D5: The SNP *whether* S denotes the sentential quantifier $WH(S)$ defined as follows: $WH(S) = \{P : P \in CNST \wedge (S \in P \vee nS \in P)\} \cup \{P \in MPP \wedge (S \in P \wedge nS \in P)\}$

In D5 the set of predicates denoted by the *whether* complement can be divided into two (disjoint) sub-sets: a set of consistent predicates and a set of inconsistent ones. Every predicate in the sub-set of consistent predicates contains the sentence S or the sentence nS and every predicate in the sub-set of inconsistent predicates contains S and nS. This sub-division is related to the distinction between indirect interrogatives which express a question and indirect interrogatives which do not express a question. Consider the following examples:

(7) Dan knows whether life is sad (or not).
(8) Dan does not know whether life is sad (or not)

Informally, only (8) is compatible with the question based on the embedded sentence; when sentence (7) is true the corresponding question in some sense does not arise.

There are two types of predicates taking *whether*-complements: responsive predicates and rogative predicates. Responsive predicates can in addition take *that*-complements. We will consider that responsive predicates, whose properties are indicated in the first part of definition D5, cannot characterise interrogatives since they cannot be inconsistent. The second part of D5 describes rogative predicates that is those which properly characterise sentences expressing questions. These predicates are strongly inconsistent and correspond to midpoint predicates such as *investigate (whether), find out (whether)*, etc. They are characterised in D6 and the interrogative complement to which they give rise—in D7:

D6: A sentential predicate P is rogative, $P \in ROG$, iff $P \in MPP \wedge P \subseteq \neg K \cap \neg K \neg$.
D7: $QWH(S) = \{P : P \in ROG \wedge \{S, nS\} \subseteq P\}$.

Thus an interrogative based on the sentence S denotes the set of rogative predicates defined in D7. Observe now that the predicate *Dan knows* is not a rogative predicate in the sense of D6 because it is not a (non-trivial) mid point predicate. Moreover, it entails *It is known.* Consequently this predicate is incompatible with any rogative predicate and this explains why (7) does not "express" a question.

Thus *yes-no*-questions are sentential quantifiers or sets of rogative predicates.

In a similar way we can analyse the constituent question (*what, who*, etc). They also denote a set of ROGs predicates:

D8: $WHO[VP]= \{P : P \in ROG \wedge \forall S \in P \exists NP(S = NP + VP) \vee S = n(NP + VP)),$ where [VP] is a set, the denotation of the verb phrase VP.

Using D7 and D8 it is easy to show that (9) entails (is set theoretically included in) (10) because every ROG predicate in (9) is a ROG predicate in (10):

(9) Who left? (10) Did Dan left?

The above proposal leads to a generalisation concerning binary sentential predicates and conditional questions. (11) should entail (12):

(11) Whether life is sad depends on whether logic is funny.
(12) Is it true that if logic is funny then life is sad.

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3rd Workshop on Relating Logic (WRL3)

Invited Lectures

Some Consideration on E. J. Nelson's Connexive Logic

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Joint work with Raffaele Mascella

Tags: algebraic logic, contra-classical logic, non-classical logic, relating semantics.

This talk will explore Everett John Nelson's connexive logic, outlined in his PhD thesis and partially summarized in his 1930 paper "Intensional Relations", which is obtained by extending the system NL (reconstructed by E. Mares and F. Paoli) with a weak conjunction elimination rule explicitly assumed in the former but not in the latter. After a preliminary analysis of Nelson's philosophical ideas, we provide an algebraic-relational semantics for his logic and we investigate possible extensions thereof which are able to cope with Nelson's ideas with much more accuracy than the original system. For example, we will inquire into extensions whose algebraic-relational semantics is endowed with irreflexive incompatibility relations, or determine a "weakly" transitive entailment. Such an investigation will allow us to establish relationships between some of the trademarks of Nelson's thought and concepts of prominent importance for connexive logic, as, e.g., Kapsner's strong connexivity and super-connexivity. Furthermore, we will show how to employ the machinery of relating logics' framework to establish relationships between some Nelsonian logics and logics which are sound and complete w.r.t. a suitable relating semantics.

If there will be time, as a further output of previous results, we will establish relationships between algebraic-relational models of Nelsonian logics and ordered structures that have gained great attention over the past years, namely partially ordered involutive residuate groupoids and (non orthomodular) orthoposets.

The Algebra of Analytic Containment

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Joint work with Damian Szmuc and Martina Zirattu

Tags: algebraic logic, philosophical logic.

We explore certain algebraic structures that naturally emerge within the framework of logics of synonymy, analytic containment, and hyperintensionality. In particular, we argue that Angell's logic AC, one of the earliest and most successful attempts to analyse the properties of logical constants with a topic-transformative character, can be better understood through a direct algebraic study of De Morgan bisemilattices. *Inter alia*, we show that a certain 9-element algebra introduced by Ferguson generates De Morgan bisemilattices as a quasivariety, making it the most adequate semantics for AC, as opposed to other 7-element and 16-element algebras considered in the literature.

Subalgebras, Topics, and Relations on Formulas

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Tags: algebraic logic, non-classical logic, philosophical logic.

Relating logics take the tools of frame semantics and adjoin to them relations on formulas in order to obtain more fine-grained models and consequence relations. This allows for the commodation of a range of non-classical logics to be semantically characterised in relating terms.

In this talk, I'll suggest an alternate avenue towards similar ends. This will proceed by way of *topics*, using my preferred modeling of such as *subalgebras* over an algebra of propositions (see [2] for more details on the modeling, and its virtues). I'll show how this machinery captures many of the features of topics that have been of interest to researchers in the past (spending some time to compare with the most popular going semantic modeling of topics, i.e. that of Lewis [1]).

The connection to relating models is simple: if we consider a class of algebras including a *free algebra* on some set of generators (we can call such a thing a *language*, for the sake of simplicity), then the subalgebras straightforwardly track the *atomic subformulas* of collections of formulas. This allows us a great deal of precision in tracking syntactic properties, and once we consider not just algebras but *matrices* (such as matrices built on algebras of a given signature which *model* a target logic), then we can use topics and subalgebras to study relations on formulas which are tracked by hyperintensional logics. I'll discuss the *Variable Sharing Property* of relevant logics and the *Proscriptive Principle* of analytic containment logics as examples.

The suggestion here is that the modeling of topics, in a way which interfaces directly with syntax, by mediation of free algebras, provides for a very simple way for experts in relating logic to apply their work to a *topic* (if you will) of current hot interest in the philosophical logic scene.

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3rd Workshop on Relating Logic (WRL3)

Contributed Lectures

On Normal Deductions for a Logic of Demodalised Analytic Implication with *Falsum*

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Tags: demodalised analytic implication, natural deduction, proof theory, relating logic, subclassical logic.

The logic of demodalised analytic implication **DAI** was first presented by J. M. Dunn [1]. And it was received by a modification of the logic of analytic (strict) implication of W. T. Parry ([6, 7], cf. [10]). Originally, the language of **DAI** consisted of the classical negation $\neg$, the classical conjunction $\wedge$, and the demodalised analytic implication $\rightarrow$. Later, R. Epstein [2, 3] described the logic **DAI** as a kind of content-relationship logic (a content inclusion logic), defining it semantically using set-assignment models. Epstein [2] was the first to consider the logic **DAI** with the constants *falsum* $\bot$ and *verum* $\top$, presenting its algebraic analysis. After that, A. Ledda, F. Paoli and M. Pra Baldi [5] analyzed the logic **DAI** with constants, also in an algebraic context, presenting the algebraic semantics for it. It should be emphasized that Epstein understood *falsum* and *verum* differently in terms of content than Ledda, Paoli, and Pra-Baldi. However, both understandings allow us to define the logic **DAI** with *falsum* semantically by set-assignment models, but using dual to each other truth-conditions for implication.

In our paper we are interested in the logic $\mathbf{DAI}_\bot$ expressed in a language without negation, but with the constant *falsum*, and in which classical negation can be defined. The aim of our paper is to discuss the problem of normalization for natural deduction systems of $\mathbf{DAI}_\bot$. And so, we will discuss two approaches to natural deduction systems and two attempts of proving the normalization theorem, in the style of G. Stålmarck [9] and in the style of D. Prawitz [8], respectively.

In the first case, for implication introduction rules, we assume certain restrictions on sets of propositional variables (see [4]):

$$(\rightarrow \mathrm{I}_1)\quad \frac{\begin{matrix}(A)\\ B\end{matrix}}{A \rightarrow B}\qquad \text{where } \mathsf{var}(B) \subseteq \mathsf{var}(A)$$

$$(\rightarrow \mathrm{I}_2)\quad \frac{E \rightarrow C \quad C \rightarrow D \quad \begin{matrix}(A)\\ B\end{matrix}}{A \rightarrow B}\qquad \text{where } \mathsf{var}(B) \subseteq \mathsf{var}(D) \text{ and } \mathsf{var}(E) \subseteq \mathsf{var}(A)$$

$$(\rightarrow \mathrm{I}_3)\quad \frac{A \rightarrow D \quad A \rightarrow C \quad \begin{matrix}(A)\\ B\end{matrix}}{A \rightarrow B}$$

$$\text{where } \mathsf{var}(B) \subseteq \mathsf{var}(C) \cup \mathsf{var}(D)$$

Let us note that rules $(\rightarrow \mathrm{I}_2)$ and $(\rightarrow \mathrm{I}_3)$ are, in fact, of a double nature. They introduce an implication, but at the same time eliminate the premises of the form of implications.

Of course, we can find such a derivation in which these premises-implications are maximal formulas. To attempt to eliminate maximal formulas, we will extend the set of presented rules by the family of new rules for implication introduction (introduction-elimination).

In the second case, we introduce auxiliary rules $(\bot^\bullet)$, $(\wedge\,\mathrm{E}_1^\bullet)$, $(\wedge\,\mathrm{E}_2^\bullet)$, $(\wedge\,\mathrm{I}^\bullet)$, $(\to\mathrm{E}_1^\bullet)$, $(\to\mathrm{E}_2^\bullet)$, $(\to\mathrm{I}^\bullet)$ that allow us to capture the constraints imposed on the set of variables from the first approach, and have only one implication introduction rule $(\to\mathrm{I})$ (see [4]):

$$(\bot^\bullet)\ \frac{A^\bullet}{\bot^\bullet}$$

$$(\wedge\,\mathrm{E}_1^\bullet)\ \frac{(A\wedge B)^\bullet}{A^\bullet}\qquad (\wedge\,\mathrm{E}_2^\bullet)\ \frac{(A\wedge B)^\bullet}{B^\bullet}\qquad (\wedge\,\mathrm{I}^\bullet)\ \frac{A^\bullet\quad B^\bullet}{(A\wedge B)^\bullet}$$

$$(\to\mathrm{E}_1^\bullet)\ \frac{(A\to B)^\bullet}{A^\bullet}\qquad (\to\mathrm{E}_2^\bullet)\ \frac{(A\to B)^\bullet}{B^\bullet}\qquad (\to\mathrm{I}^\bullet)\ \frac{A^\bullet\quad B^\bullet}{(A\to B)^\bullet}$$

$$(\to\mathrm{I})\ \frac{\begin{matrix}(A) & (A^\bullet)\\ B & B^\bullet\end{matrix}}{A\to B}$$

Additionally, we will present a new rule that allows for the combined use of the elimination and introduction rules of logical constants and our auxiliary rules. This new rule will enable us to derive counterparts of rules $(\to\mathrm{I}_2)$ and $(\to\mathrm{I}_3)$ with no explicit restrictions on the sets of variables and to attempt to "atomize" the *reductio ad absurdum* rule.

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Presupposition and Relating Semantics

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Tags: presupposition, relating conjunction, relating semantics.

In my talk, I will discuss the application of the general idea of relating semantics to presupposition projection, restricting the consideration to the case of conjunction. As an illustration of the topics I will be addressing, let us consider the following example of a conjunction in which the concept of presupposition is essential.

(17) (a) John stopped drinking and Anne is no longer complaining.
(b) John drank and Anne complained.
(c) John drank and that's why Anne was complaining.
(d) Anne is no longer complaining and John stopped drinking.
(e) Anne complained and John drank.
(f) Anne complained and that's why John drank.

Pretheoretically sentence (17a) has the presupposition (17b). This presupposition is related to the occurrence of "stopped" (in the first conjunct) and "no longer" (in the second conjunct) phrases in (17a). However, utterance (17a) would be quite confusing if both conjuncts were not content-related. Pragmatics teaches us that the meanings of component sentences are most often linked together (and if not, another meaning appears). So, it can be assumed that there is a connection between John's drinking and Anne's complaining, i.e., Anne complained that John was drinking. Then, however, the presupposition of the sentence (17a) is the proposition (17c). The application of the relating relation idea can be seen here: the truth of a judgment (17b) in which there is "and" does not come down to classical truth-conditions, but requires that the causal relation (and time sequence) be taken into account. The converse of this conjunction illustrates this well, i.e., the sentence (17d), which has a presupposition (17e), but after taking the causal relationship into account has a presupposition (17f).

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Relating Logic with Negative Relational Properties

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Joint work with Tomasz Jarmużek and Mateusz Klonowski

Tags: non-classical logic, philosophical logic, relating logic.

This talk shows how to incorporate negative relational properties into Boolean logic with relating implication. I will show the limitations of existing systems that only handle positive relational properties like "A relates to B", and explain why applications require explicit negative statements such as "A does not relate to B". The main focus will be on extending Algorithm α, the method for transforming relational properties into axioms, to handle negative cases through new transformation functions α^{-}. I will present concrete examples showing how negative inference rules (Rα^{-}) work alongside existing positive rules. The presentation concludes with soundness and completeness results for the extended system.

Some Algebraic Insights About Epstein's Logics

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Tags: algebraic logic, non-classical logic, philosophical logic.

Epstein's logics are a family of logical systems developed by Richard Epstein starting with the seminal paper [1], reaching a fully general theory in the monograph [2], originally published in 1990. Among the plethora of logics recaptured by Epstein's semantic investigation, I focus on the original families of relatedness and dependence logics. The basic intuition behind Epstein's programme ([6]) is that the logically significant properties of a sentence are not only its truth-value but its content as well (in this sense Epstein's work is as a precursor of part of the current discussion about hyperintensionality and, in particular, the so-called two-component semantics).

There are two options in Epstein's framework about how to formally represent the content of a sentence. The first is a relation-based approach, in which two formulae are related ($R(\alpha, \beta)$) when their contents are (the reading of the relation is left to the single logics, e.g. inclusion in one direction). The relational approach has been undergoing an intense renaissance within the much vaster research programme of relating logics, advanced by the Torunian logic group (see [3],[4],[5]), greatly expanding the scope of Epstein's original work. The second approach is function-based and is known as set-assignment semantics: here variables are assigned, besides their extensional truth-value, a content in the form of a subset of a certain fixed countable set U via an assignment function $s : Var \to \mathscr{P}(U)$. Complex formulae receive their content recursively simply by forming the union of the contents of their variables (the map is called a union set-assignment). The standard valuation function operates as usual over Boolean connectives, while the arrow $\to$ receives an intensional reading: $\alpha \to \beta$ is evaluated as material implication if a certain relation holds between the content $s(\alpha)$ of the antecendent and that of the consequent $s(\beta)$. In the case of dependence logic $\mathbf{D}$, said relation is $s(\alpha) \supseteq s(\beta)$.

Set-assigment semantics is a versatile tool, in fact by tweaking the conditions on s we can obtain classes of models which characterize well known logics, like intuitionistic logic, various modal logics, many-valued logics like Łukasiewicz and strong Kleene logics. Despite all of this, the inner workings of Epstein's systems are concealed by a redundant structure, which, once seen in its more essential algebraic components, allow set-assignment semantics to reach a higher degree of versatility.

In the specific case of models for dependence logics, we are working with two algebras, the 2-element Boolean algebra in order to assign truth-values to formulae, and a second algebra (in this example $\mathscr{P}(U)$) which provides formulae with their intensional content, but even here we don't need the full powerset structure, just its join-semilattice reduct, since the content of complex formulae is obtained by set union. Two mappings assign formulae to, respectively, their extensional and intensional values. What was left implicit in Epstein's work is a translation from the language in the type of Boolean algebras to that of set theory.

The common structures underlying Epstein's investigation are models for a language of type ρ_A of the form $\langle \mathbf{A}, \mathbf{B}, N, v_A, v_B \rangle$ where $\mathbf{A}$ is an algebra of type ρ_A and $\mathbf{B}$ is an algebra of type ρ_B. $N : Fm_{\rho_A} \to Fm_{\rho_B}$ is a translation which preserves variables, that is to say $N(x) = x$, and it is closed under substitutions, that is $N(\alpha(x_1/\beta_1, \dots x_n/\beta_n)) = N(\alpha)(x_1/N(\beta_1), \dots x_n/N(\beta_n))$ for all variables $x, x_1, \dots x_n \in Var$, formulae $\beta_1, \dots \beta_n \in Fm_{\rho_A}$ and connective $\alpha \in \rho_A$. Finally $v_A : Fm_{\rho_A} \to A$ and $v_B : Fm_{\rho_B} \to B$ are homomorphisms except at most for a certain fragment of the language which contains the properly intensional connectives (in the case of dependence logic, only $\to$ belongs to the intensional language). The two universes A and B can be intended as respectively the set of extensional and intensional (namely the content) values which formulae can take.

Within this generalized setting we can straightforwardly rephrase dependence models, obtaining an equivalent complete semantics for the various dependence logics **D**, **dD**, **Eq**, and by putting as algebra of contents the entire lattice of theories of classical logic also the classically dependent logic **DPC** is obtained. Furthermore it is possible to recapture relatedness logics as well, both in their symmetric (**S**) and non-symmetric (**R**) versions. Both cases show how in these logics the structure of content is implictly assumed to be that of a lower bounded distributive lattice, which provides the algebra of contents for relatedness models (although a finer content assignment function is required in the particular case of **R**).

The main point of this generalization of Epstein's set-assignment semantics is not just to rephrase already known semantics in new terms, but to show how a deeper algebraic understanding of Epstein's systems can allow for a more general semantics which can be employed—in a similar fashion as relating logics did successfully—to systematically describe logics beyond the original scope of Epstein's investigation, which was done in a first case study for Parry's logic of analytic implication ([7]).

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1st Workshop on Mechanisms and Causes (WMaC1)

Invited Lectures

Interventionism and Causal Reasoning: Challenges and Possibilities

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Tags: philosophy of science.

There is no question that causal reasoning plays a significant role in scientific practice. Whether one is considering the role of greenhouse gases, such as CO_2 and methane, in global warming, or trying to identify causes of certain symptoms in patients in medical diagnosis, causal reasoning is clearly employed. But what is the structure of such reasoning? James Woodward offers an insightful answer to this question by identifying the crucial role played by interventions. On his view, two variables are causally related provided that interventions in one variable changes the other, while the causal relations among them remain invariant under the interventions.

The interventionist approach allegedly faces a number of challenges: (i) It is circular, since the notion of intervention is itself causal. (ii) The approach makes causation depend on human agency, since interventions depend on actions and intentions of agents (Price). (iii) The interventionist approach is ultimately dispensable, since familiar counterfactual approaches manage to accommodate the same issues accounted for by interventionism but without invoking interventions (Reutlinger). (iv) The approach is unable to distinguish correlation (which is symmetric) from causation (which is asymmetric) in the presence of certain laws. (v) It is unclear how the approach can accommodate cases that involve causal processes despite the impossibility of any intervention in the relevant systems (e.g., in astronomy) or cases that do not involve causal relations despite the presence of interventions (such as Einstein-Podolsky-Rosen (EPR) setups in quantum mechanics).

In this talk, I offer a defense of the interventionist approach within a thoroughly modalist empiricist view.

Thinking About Pathways and Mechanisms in Chemistry and Biology

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Tags: philosophy of science.

What is the relation between the notions of mechanisms and pathways in scientific practice? While for some philosophers these notions are not be viewed as really distinct, for others they have important differences. The aim of this talk is to revisit this issue, by examining the notions of mechanism and pathway in chemistry and biology. It will be argued that there is no clear distinction to be found in scientific practice between mechanism-talk and pathway-talk. By focusing on examples from chemistry and biochemistry, I will argue against the proliferation of causal notions in scientific practice. I will claim that, in reality, the notions of mechanism and pathway are instances of the same general notion, that of a causal process or chain. My strategy will be to discuss various ways that the distinction between mechanisms and pathways can be established, and use specific chemical and biological examples to show that no clear distinction between the two notions can be made by close inspection of how the notions are used in scientific practice.

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Constitution and Causation in Models of Mechanisms

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Tags: metaphysics, philosophy of science.

Craver [1] has argued that constitutive relevance can be discovered by mutual manipulability, based on interventions. However, the requirements on interventions make mutual manipulability of mechanisms and their constituents impossible.

Models of multi-level mechanisms can be constructed on the basis of empirical information although the relevant experiments directly provide only information about causal relations, because this causal information can bear on variables at different levels. A multi-level model is built in two steps: 1) first, partial purely causal models are built for each hypothetical constituent variable Φ_i, on the basis of top-down and bottom-up experiments that modify or measure Φ_i in a level-specific way, 2) second, those partial models are merged in a comprehensive model containing both causal and constitution relations between variables, on the basis of information about the level of each variable and spatio-temporal constraints.

Craver, Glennan, and Povich's thesis (see [2]) that constitutive relevance can be reduced to "causal betweenness", between the input and the output condition of a mechanism, is not adequate for multi-level mechanisms. Their account of the construction of models for mechanisms leads to the paradoxical result that there are no levels and no multi-level mechanisms, but only causal chains of fundamental level activities.

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On the Possibility of Inconsistent Causal Explanations

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Tags: causal explanation, Hawking's radiation, incompatible theories.

Philosophical accounts of scientific explanation often treat consistency as a non-negotiable epistemic value. In particular, causal explanations are frequently assumed to derive their legitimacy from their coherence with accepted background theories [1, 2]. According to this view, inconsistency undermines not only the truth of an explanation but also its explanatory force.

Yet, certain kinds of inconsistency may leave explanatory and predictive power intact, or even, in some cases, enhance it. For instance, the standard causal explanation of Hawking radiation serves as a compelling instance of the value of non-consistent explanations. In this case, particle–antiparticle pairs form near a black hole event horizon, one falls in, one escapes as radiation—appealing to both quantum field theory and general relativity. These theories are known to be in tension (indeed inconsistent) in certain regimes. Nevertheless, the explanation remains deeply epistemically robust: it facilitates intuitive understanding of black hole thermodynamics, is employed in pedagogical settings, inspires predictions (e.g. about black hole evaporation), and influences theoretical work seeking quantum gravity. The way this causal picture retains fruitfulness despite its inconsistency

I will defend the claim that causal explanation can legitimately be explanatory even when inconsistent. My argument proceeds along two complementary fronts. On the one hand, I show how such explanations may preserve a range of epistemic virtues often thought to depend on consistency—including empirical accuracy, predictive fertility, heuristic and organizational power, and intuitive accessibility. On the other hand, I argue that, although rare, inconsistent causal explanations can also display the very features traditionally associated with explanatory success in the Hempelian tradition, namely predictive power and the promotion of understanding.

In order to make this case, I proceed in four steps. First, I examine the commonly assumed tight connection between causal explanation and logical consistency. Second, I challenge this assumption by presenting the standard account of Hawking radiation as an instance of a theoretically inconsistent yet successful causal explanation. Third, I explore the contexts in which such explanations may arise and even flourish. In this discussion, I show how they can preserve a range of epistemic virtues often thought to require consistency, while also achieving predictive power and the promotion of understanding. Finally, I reflect on the frequency and risks of these explanations, arguing that in a scientific landscape where data collection increasingly outpaces unificatory theory, we should expect such cases to become more common.

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Causes for Indeterministic and Non-Local Contexts

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Tags: philosophy of science, temporal logic.

This talk develops a theory of *causae causantes* initiated by Belnap [1] and extended by Belnap, Mülller, and Placek [2], with an aim to show how it applies to indeterministic and non-local contexts. The background for Belnap's theory is a rigorous framework of concrete events, like Branching Space-Times (BST) of Belnap et al's [2] or Müller's [4] theory of transitions. A transition is understood as a pair of concrete events, just one event after another event. A basic transition is a point event and a possible continuation of it. Of particular interest are alternative basic transitions: they share an initial event, but have different possible continuations. Transitions are important for Belnap's indeterministic theory. They figure as the both relata of the causal relation: the objects that are caused are transitions and the objects that cause them, their *causae causantes*, are (basic) transitions. This is supported by Belnap's [1] formal results that causae causantes of a given transition satisfy the (generalized) INUS condition of Mackie [3]. As for nonlocality, in this talk I focus on non-probabilistic quantum cases, modelled after the EPR correlations and GHZ setups. In such nonlocal correlations, a transition from a measurement event to an outcome event has at least one causa causans that does not lie in the past of the outcome event in question [2]. I will investigate the formal properties of such "badly-located" causae causantes.

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A Model for Computing Probabilities of Complex Conditionals

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Joint work with Anna Wójtowicz

Tags: conditionals, general methodology of science, probabilistic explanation.

Conditionals stand in a systematic relation to the notions of causation and explanation. They are of particular importance in discussions of probabilistic explanation, insofar as a substantial class of conditional statements both presuppose probabilistic concepts and admit of a probabilistic construal. What is therefore required is a rigorously specified framework that enables the principled assignment of probability values to conditionals.

For simple conditionals, the situation might appear straightforward: it is often taken for granted that the probability of a conditional of the form $A \to B$ is to be identified with the corresponding conditional probability $P(B|A)$ defined over some underlying probability space $S = (\Omega, \Sigma, \mathrm{P})$ (P being the known probability distribution). Yet even this identification might be considered *ad hoc*, and needs to be justified in a systematic way.

Matters become even more complex in the case of higher-order constructions—such as conjoined conditionals $(A \to B) \wedge (C \to D)$ or conditional conditionals $(A \to B) \to (C \to D)$ (or the simpler, yet important right-nested conditionals $B \to (C \to D)$). They deserve attention as they express natural claims in science or everyday life. In these cases, the primary theoretical challenge is to articulate a systematic procedure for computing their probabilities, which includes the construction of the relevant probability space.

The main aim of the talk—besides addressing philosophical and methodological questions—is to describe a general model for computing probabilities of conditionals. The mathematical framework is provided by the theory of Markov chains (only rudimentary facts are really needed). The conditionals are represented as games and their probabilities as probabilities of winning the game. The mathematical counterpart is the absorption probability in the distinguished winning state in the constructed graph $G(\alpha)$ (corresponding to the conditional α). We restrict our attention to conditionals where the antecedents have a positive probability.

In the talk an inductive construction of a family of graphs $G(\alpha)$ is presented. Each graph $G(\alpha)$ is designed to model a particular conditional α and it generates a canonical probability space $S(\alpha) = (\Omega(\alpha), \Sigma(\alpha), \mathrm{P}(\alpha))$. In this space α is given an interpretation as an event $[\alpha] \subseteq \Omega_\alpha$ so that the probability of α can be computed as $P_\alpha([\alpha])$. (This also defines a semantics for conditionals, which opens way to discuss logical and metalogical issues). The graph allows to compute this probability by solving a system of linear equations. In order to give a general inductive definition of the graph $G(\alpha)$ we define three operations on graphs which correspond to the negation, conjunction and the conditional operator $\to$.

The definition enables the construction of graphs and the corresponding systems of equations for arbitrarily complex conditionals in an algorithmic and efficient manner.

1st Workshop on Mechanisms and Causes (WMaC1)

Contributed Lectures

Molecular Mechanisms: Epistemological Aspects of MD Simulations

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Tags: philosophy of science.

In this talk, we focus on explanatory practices in the field of molecular dynamics (MD) simulations, with particular attention to the role of epistemic strategies—such as abstraction, idealization, and approximation—in studying biological structures like proteins and their molecular mechanisms. Specifically, we examine a subclass of idealizations called as defective idealizations. We analyze key theoretical and experimental aspects of MD simulations, with an emphasis on Gō-like models, highlighting how various epistemic strategies are employed not only in model construction but also in shaping experimental design and guiding the validation process.

On the one hand, we draw broader philosophical conclusions about the explanatory power of MD simulations in advancing scientific understanding of underlying causal processes; on the other hand, we highlight the limitations of mechanistic explanation within this investigative context. Finally, we offer a broader philosophical analysis of the simulation process, arguing that our account departs from the overly rigid explanatory frameworks advocated by some proponents of the new mechanistic philosophy.

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Fundamental and Non-Fundamental Causation in Russell's Metaphysics of Science

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Tags: causation, Russell, epistemic structural realism, space-time.

Bertrand Russell is well known for his causal eliminativism in his celebrated *On the Notion of Cause* (1912), a view he famously expressed by his claim that causation is retained as an important scientific notion only because, like the monarchy, people assume it to do no harm.

On this view metaphysically fundamental scientific theories, such as mathematical physics, dispense with the notion of cause in their fundamental physical laws. However, years later in *The Analysis of Matter* (1927) and *Human Knowledge* (1948) Russell returned to the discussion of causation and argues for the following two views, which are *prima facie* incompatible with each other and incompatible with his earlier claim. First, that there are separable causal lines which should be identified with physically distinguished geodesics in relativistic space-time (1927, 1948) and that, (2) causation is a fundamental postulate in non-demonstrative scientific inference (1948) in his epistemic structural realism. These two claims seem inconsistent with Russell's main claim in *On the Notion of Cause*, but in this talk I will argue, after elaborating on these theses, that these are compatible with each other and with Russell's earlier eliminativistic thesis. Furthermore, I explain how these are part of the logical atomist research program (Landini, 2014; Klement, 2017; Elkind, 2019).

Causal Coherence and Defeasible Reasoning: Toward Adaptive Logics for "Because"

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Tags: adaptive logics, history of logic, non-classical logic.

Learning to recognize and construct causal explanations is a fundamental part of how both children and scientists come to understand the world. Consider a classroom setting where a teacher drops a pen and comments, "Objects fall because there is a force called gravity that pulls them down". Here, an everyday event is explicitly connected to a general principle, inviting the learner to see the observation as part of a broader explanatory pattern. When the same child later watches a piece of ice float in a glass of water and asks, "Why doesn't it sink?", the adult's reply—"It is less dense than water"—does not include an explicit *because*. Even so, it clearly prompts the child to infer the underlying causal relation: anything less dense than water floats on water. In both cases, understanding arises not merely from the exchange of individual statements, but from the child's ability to construct *bridging inferences*—context-sensitive connections that integrate what is said into a coherent explanatory structure.

What these examples highlight is not just the use of causal language, but the listener's capacity to rely upon what Hobbs in [3] called *coherence relations*—in this case, explanatory ones. Whether marked explicitly with a connective like *because* or left implicit, such relations guide the integration of information into a larger conceptual framework. Among the many types of coherence relations—temporal, contrastive, elaborative—it is the *explanatory* and *diagnostic* varieties that are most closely associated with causal reasoning.

This paper focuses on causal connectives—such as *because*, *since* and *as*—as surface cues that signal the presence of these inferential structures. Building on insights from psycholinguistics and formal semantics, I develop a formal framework that models the inferential patterns these connectives support, with special attention to their context-sensitive and defeasible character.

To illustrate these patterns, I examine contrastive examples where *because*-statements support different types of inferences. In explanatory cases such as "There are holes in Ann's clothes, because there are moths in her wardrobe", the *because*-clause identifies a cause. In diagnostic cases like "There must be moths in Ann's wardrobe, because there are holes in her clothes", the same connective marks an inference to the best explanation. While both rely on coherence, their inferential direction and pragmatic function differ.

I argue that *because*-statements of the diagnostic type can be modeled as summaries of underlying abductive inferences, which may be accepted tentatively and later revised in light of further evidence. This motivates a formal treatment within the framework of adaptive logics, which provide a unified formal framework to model defeasible forms of reasoning (see, for instance, [2]). The resulting system draws on Rott's account of difference-making conditionals [4], and connects with recent developments in the semantics of counterfactuals—particularly Santorio's dynamic framework, which emphasizes the role of background assumptions and context change [5].

While the proposed framework draws on recent developments in logic and semantics, it also resonates with earlier traditions—particularly the work of the Lvov–Warsaw School. Though they did not focus on causal connectives *per se*, several figures within the School anticipated key aspects of the present approach. For instance, Ajdukiewicz's model of subjectively uncertain inference [1] reflects the kind of defeasible reasoning characteristic of diagnostic uses of *because*, where the conclusion is accepted with less confidence than the premises. His earlier defense of the equivalence between indicative conditionals and material implication [7] likewise demonstrates an interest in how natural language conveys logical relations. In a different but related vein, Twardowski's critique of "symbolomania and pragmatophobia" [6] underscores a shared commitment to grounding logical analysis in communicative and inferential practice.

Taken together, the insights developed in this paper aim to bridge a gap between formal logic and the subtle inferential practices found in natural language explanation. By accounting for the context-sensitive and defeasible character of coherence relations—particularly in the case of causal connectives—the proposed framework captures aspects of reasoning that standard truth-functional approaches tend to overlook. It not only clarifies the structure of explanatory and diagnostic uses of *because*, but also shows how such structures interact with broader patterns of belief revision and counterfactual reasoning. In doing so, it contributes to a growing body of work that seeks to align logical models more closely with the cognitive and communicative processes that underpin explanation, both in scientific discourse and in everyday understanding. This perspective also resonates with certain strands in the Lvov–Warsaw School, particularly in the work of Ajdukiewicz and Twardowski, where attention to linguistic practice and inferential behaviour complemented formal rigour.

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Causal Semantics of Concurrent Systems

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Joint work with Ryszard Janicki, Jetty Kleijn and Maciej Koutny

Tags: set theory and the foundations of mathematics, causal structures.

Concurrent systems can be formally described and analysed at different levels of abstraction: from individual behavioural observations to concurrent histories represented by causality structures capturing intrinsic dependencies between event occurrences, to system level models. When developing an operational semantics of such systems, one needs to choose a suitable notion of execution model, e.g., by adopting a class of partial orders according to which event occurrences are arranged. Typical kinds of such partial orders are the total, stratified, and interval orders, and we will investigate the key implications of selecting each of these three execution models.

Analysing the vast number of executions of concurrent system is impractical. A more effective way is to consider behavioural specifications based on intrinsic relationships between events (such as those represented by causal partial orders) resulting in different classes of relational structures, where each such structure covers a (large) number of individual executions. The relational structures considered here are based on two relationships between events—the 'before' and 'not later' relationships—which can be used to express and analyse causality, independence, and simultaneity between events. We will also describe language-theoretic devices corresponding to the different classes of partial orders and relational structures.

Our discussion is based on a view that to design a sound semantics of a class of concurrent systems, one needs to provide: (i) a suitable formal model of the executed behaviours; and (ii) a support for effective analytical techniques of system behaviour. Typically, (i) involves choosing a class of partial orders (e.g., total, stratified, or interval) according to which event occurrences are arranged, and (ii) can be achieved by introducing suitable relational structures which are more abstract, and each such structure can cover a (large) number of executed behaviours. Moreover, such relational structures usually capture intrinsic relationships between executed events, and can be used to express and analyse causality, independence, and simultaneity.

Formal models of operational semantics of concurrent systems are typically based on models for representing individual system behaviours, where two events (executed actions) can be observed as either happening one after another or simultaneously. Such behaviours can be represented as partially ordered sets of events where execution precedence is transitive, and simultaneity can be captured by a lack of ordering. In the literature, one can find three main kinds of partial orders modelling concurrent behaviours: (i) total orders modelling sequential executions of individual (instantaneous) actions; (ii) stratified orders modelling sequential executions of sets of simultaneous (instantaneous) actions; and (iii) interval orders modelling behaviours where events can take time and the corresponding time intervals overlap arbitrarily.

Dealing directly with the vast number of actual behaviours of a concurrent system is far from being practical. It was realised long time ago that it is more effective to discuss behaviours at more abstract level of behavioural specifications (often based on intrinsic relationships between events such as those represented by causal partial orders), each such specification—typically, a relational structure—encompassing a (large) number of behaviours.

During the operation of a concurrent or distributed system, system actions may be executed sequentially or simultaneously. To faithfully reflect the simultaneity of actions, one may choose step sequences (or stratified orders of events) where actions are regarded as happening instantaneously in sets (called steps), or interval orders where actions are executed over time intervals.

Dealing with the semantics of concurrent systems purely in terms of their individual executions, such as total orders (sequences), stratified orders (step sequences), or interval orders is far from being computationally efficient in terms both of behaviour modelling and of property validation. To address this shortcoming, more involved relational structures have been introduced, aiming at a succinct and faithful representation rs of (often exponentially large) sets RS of closely related individual executions. Examples include causal partial orders for sequences, and invariant structures for step sequences. Succinctness is usually achieved by retaining in rs (through intersection) only those relationships which are common to all executions in RS. Faithfulness, on the other hand, requires that all potential executions which are extensions of rs belong to RS. Structures like rs, referred to as invariant structures represent concurrent histories and both desired properties (succinctness and faithfulness) follow from generalisations of Szpilrajn's Theorem (i.e., a partial order is the intersection of all its total order extensions). Although rs provides a clean theoretical capture of the set RS, to turn them into a practical tool (as, e.g., in [31]) one needs to be able to derive them directly from single executions using the relevant structural properties of the concurrent system. This brings into focus relational structures with acyclic relations on events (e.g., dependence graphs introduced in [33] and analysed in detail in [9]), which yield invariant structures after applying a suitable closure operation (e.g., the transitive closure for acyclic relations).

The approach sketched above has been introduced and investigated in [11, 12] as a generic model that provides general recipes for building analytic frameworks based on relational structures.

Related work Using relational structures to model concurrent behviours in the presented way was initiated in the late 1980s [14, 15, 16, 6, 29]. More recently, the approach have been substantially revised and generalised first in [10] and then in [11, 13, 12].

The language-theoretic framework for abstract captures of concurrent histories—called traces—have been introduced by Cartier and Foata in 1969 [1], and independently reinvented by Mazurkiewicz in 1977 [32] with a different motivation to provide a tool to model causality. Dependence graphs have been introduced in [33] and analysed in [9]. An extensive account of trace theory is provided by [4]. Applications of traces in combinatorics can be found in [1, 2, 5]. The COSY [18] model of concurrency assumes that concurrent systems are compositions of sequential components, and to capture this semantically [35] introduced the concept of vector firing sequences. It turned out that traces and vector firing sequences can be considered as different representations of the same kind of partially commutative monoids [18]. There are also several strands of related research on traces which have not even been mentioned, e.g., infinite traces [7, 8, 28]. And algebraic properties of traces can be found in, e.g., [4, 3].

The development of (finite and infinite) process semantics based on stratified order structures for different classes of Petri nets with context arcs was presented in, e.g., [21, 22, 23]. The treatment was also extended to other models of concurrent systems in, e.g., [19, 25, 26, 27].

For the model of step traces, [34] introduced and applied the notion of indivisible steps, the lexicographical form of step traces, as well as the representation of a step trace utilising its linear projections to binary action sub-alphabets. Also, it solved in an efficient way, the problem of step sequence equivalence in the context of step traces. Moreover, [30] provides and analyses a detailed representation of step traces by the stratified order structures and combined dependency graphs of [24].

The paper [20] deals with the algebraic properties, such as projections, hiding, and form, of step traces and interval traces (where the structures underlying observation are interval orders). In [17] a representation of interval orders by sequences of antichains is discussed.

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Neuroprosthetics and the Mechanism of Mind-Body Causation

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Tags: causal exclusion, causation, control theory, dualism, identity theory, epiphenomenalism, multiple realisation, physicalism, reductionism, supervenience.

The central question of the philosophy of mind is arguably the question of how to account for the apparent causal influence of the mind to the body, and vice versa, and how to account for the apparent power of our mental states—beliefs, desires, emotions and so on—to have a causal influence on the physical world surrounding us: how can minds, qua minds, bring about physical changes, including changes in our bodily movements.

In the recent philosophy of mind, this issue has been discussed in terms of the causal exclusion problem: assuming (as the physicalist worldview does) that the physical world is causally complete—that every physical event that has a cause has a complete physical cause—mental causes are left with a surplus role, at best, in accounting for the physical courses of events [1, 2]. In the most recent developments, the analysis of this problem has been focusing on the notion of causation at play in the argumentation, and it has been claimed that if we adopt a more modern, scientifically informed view on causation, the problem evaporates, and we can see how mental states can be genuinely and fully causally efficacious.

These arguments have drawn explicit inspiration from research on neuroprosthetics, in particular from the fact that modern neuroprosthetics have moved beyond being mere motoric devices (connecting to the motor cortex), and are now functioning on the basis of the intentions of the patient (connecting to the neural correlates of intentional action). This vividly demonstrates, the claim is, how mental states—of which intentions to act are a prime example—can have an effect on the physical courses of events ([3, 4, 7, 8, 9]. However, a more detailed look at this argumentation reveals that it rests on a misinterpretation of the neuroscience involved: it is true that there is such a variation at the level of the activity of single neurons that this cannot be identified as the basis of intentional action, but a relevant neural constant can nevertheless be identified at the level of ensembles of neural activity [6]. Therefore, neuroprosthetics is a demonstration of physical-to-physical causation (from neural ensembles to bodily/robotic movement), not mental-to-physical causation; it is a demonstration of the mind-body identity theory.

Here, I will simply assume all of the above. None of it should be particularly controversial. Instead, the really interesting questions are the ones that all this prompt. Most importantly: how do we identify mental states (intentions to act) as certain neural states (ensembles of neural activity); what makes us establish such a link between the mental variables and the neural variables? I will here outline an answer to these questions.

Note, first, that these sorts of questions are rarely voiced. There are good reasons for this: there are no formal rules for identity introduction. In all formal systems identity can be introduced only reflexively, with regard to entities themselves (i.e. "from a one can infer $a = a$"). But nobody has issues with self-identity; what we want to know is how different things can be deemed to be the same. As the current debate on the mind-body problem converges on the notion of causation, and the arguments defending the idea of mental causation appeal to precisely defined accounts of causation, this issue becomes very tangible: neither causal modelling (interventionism) nor modal logic (difference-making) provides criteria for identity introduction.

In neuroprosthetics, however, we appear to have a concrete, pragmatic example of identity introduction—the whole research paradigm is based on the idea that we can link mental states with their neural basis: "if an individual has two potential reach goals, an apple and an orange, and the subject prefers apples over oranges, there are signals in his or her brain that indicate this preference and will influence the decision to reach for the apple instead of the orange" [5, p. 258]. The assumption is that something neural will always correspond to our mental states.

So, how do neuroprosthetics link the mind and the brain? The core of neuroprosthetics is in engineering, more precisely in control theory: the aim is to build a device that connects, as accurately as possible, the values of one variable to the values of another variable (e.g. the values of the function of a thermostat to the values of a room temperature). One starts by defining a target (a process variable), in relation to which we can assess the level of efficacy of the given causes (control variables). The next step is to establish a feedback loop between these entities (closed loop control system): the connection between the target and its cause must be as stable as possible (which can be achieved by tuning the value of the control variable in response to the values of the process variable). Basically, neuroprosthetics comes down to defining a control system, encompassing of two subsystems (controller system and target system), and establishing a stable link between these two (changes in one state space connect reliably with the changes of the other).

Defining the targets of control in neuroprosthetics is relatively easy (targets of control are typically easy to define): it's the given motor task. The difficulty is to find the best (most stable) way to control these targets. This boils down to the computational task of identifying constants from large neural data sets. How does one identify such constants? This is the crux of the matter: these are constants only in relation to the given target. So, we do not identify the neural correlates of intentional action in vacuo (as the philosophical mind-body debate easily suggests), but always in relation to a specific target.

What can neuroprosthetics teach us about the mind-body problem? There are two lessons. First, it is indeed the case that we should view the mind (mental causation) identical with the brain (physical causation). However, this talk makes sense only in a context where the relevant variables have been precisely defined. This is something that neurosprosthetics can provide. Second, it's an open question how far we can stretch this method of analysis. We are still working in the confines of relatively simple actions. Once we add complexity to the tasks, by adding cultural variation in the target settings, for example, we might have to expand the domain of entities we need to consider in defining the best control. This might take us beyond the neural, as we might have to take into account complex interactions brains (people) and their surroundings, but not beyond the physical, as brains are a proper subset of the physical world. This might lead us to conclude that mental states are not identical with brain states after all, but only with more encompassing physical states. This equivocation between mind-brain and mind-physical reductionism could explain why, regardless of the wide-spread acceptance of metaphysical physicalism, debates regarding the mind-body problem keep on going.

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Balancing the Scales: Predictive Processing Between Mechanistic and Normative Explanations

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Tags: mechanistic explanations, philosophy of cognitive science, philosophy of science, predictive processing.

Predictive Processing (PP) is a theoretical framework that bridges two distinct approaches in understanding brain function: one conceptualizing the brain as a Bayesian probabilistic machine (cf. [15, 19]), and the other emphasizing unconscious perceptual inference (cf. [10, 7]). Proponents of PP argue that, regardless of the specific version, the framework aims to provide a computational model of cognitive mechanisms based on precision-weighted, hierarchical, bidirectional message passing and error minimization (cf. [3, 11]). In this view, the brain encompasses a multi-level, hierarchical generative model of its environment. In this context, a critical question arises regarding the explanatory status of PP as a theory that aims to account for (at least some) cognitive, perceptual, or motor functions. Many researchers defend the view that PP provides a sketch of the mechanism, in which some structural aspects of a mechanistic explanation are omitted—leaving out crucial details about how the mechanism works. Such sketches are incomplete or elliptical representations of target phenomena (explananda) (cf. [20]). Consequently, explanations provided by PP are often considered mechanistic (cf. [1, 6, 8, 9, 12, 13]). However, there remains ongoing debate about how exactly to interpret the mechanistic sketch in relation to predictive architecture? However, the claim of PP's mechanistic nature is contested by several scholars (cf. [2, 5, 14, 17, 18]), who argue that Bayesian frameworks like PP cannot be directly linked to causal-mechanical explanations, as they are driven by the mathematical formalism of Bayesian methods rather than causal hypotheses regarding how mechanisms give rise to specific phenomena. As such, mechanistic interpretations of PP may be unjustified. Given these debates, I propose an alternative perspective — I claim that mechanism sketches, as employed within the PP framework, should be reconceptualized as comprising two irreducible components: the mechanistic, which captures the structural and causal organization of the system, and the functional, which introduces explanatory constraints that govern cognitive performance under conditions of uncertainty and limited resources. On this view, PP models identify explanatory constraints that are inherently linked to a normative mode of explanation. By explanatory constraints, I refer to the formal, structural, and functional limitations that affect how theoretical and formal models can represent, simulate, and explain neural and cognitive processes (cf. [21]). These constraints arise from both the biological properties of the nervous system and the mathematical and physical principles governing information processing. These constraints determine how explanations must be constructed, considering both adaptive mechanisms and the integration of perception with action. In developing this account, I draw on Levenstein et al. [16], who argue for a pragmatic reinterpretation of cognitive neuroscience. On their view, mechanistic, normative, and descriptive theories each have a legitimate role, with their use determined not only by structural accuracy but also by epistemic utility and relevance to specific research aims. This pluralistic framework enables integration across explanatory levels without reducing one mode of explanation to another, while allowing for the development of more adequate future models.

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Mechanisms from a Psychological Standpoint

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Tags: cognitive psychology, mechanisms, philosophy of science.

The concept of mechanism is central in cognitive psychology, while also being at the forefront of discussions in the philosophy of science. This talk explores the concept of mechanism in cognitive psychology in light of current debates in philosophy. Philosophers of science have distinguished between ontological and methodological approaches to mechanisms [5]. In cognitive psychology researchers refer to mechanisms that support the development of cognition from infancy to maturity [4, 2]. Mechanisms reveal how learning is possible, but also provide us with a framework about what concepts are fundamental for understanding how learning works.

In psychology we find a very "thin" definition of a mechanism: "in general, a device or physical property by which something is accomplished, or an explanation that relies on such a device or property" (APA Dictionary of Psychology). On the other hand, the philosopher of science who have studied philosophy of mechanisms in biological sciences have offered more "thick" descritpions for what a mechanism is, as in [3]: "mechanism for a behavior is a complex system that produces that behavior by the interaction of a number of parts, where the interactions between parts can be characterized by direct, invariant, change-relating generalizations". The purpose of this presentation is to compare and contrast the concept of mechanisms in the fields of psychology and molecular biology and neurobiology. We will look into the "Quinian bootstrapping mechanism" proposed by Susan Carey [1] as a case study of a learning mechanism. We will address questions about how the term mechanism is (minimally or "loosely") applied in psychology in comparison to standard characterizations of mechanisms in philosophy of science, which entities (procedures or states) count as components of a mechanism, to what extent they are causally linked, and how is a causal connection between the components identified. We will further investigate the role mechanism plays in cognitive psychology: when psychologists talk about mechanisms do they commit themselves to mechanistic explanation and what would it mean for psychological explanations to be "mechanistic".

The central claim of this talk is that the concept of mechanism is very different in cognitive psychology from the concept of mechanism in the philosophy of biological sciences.

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1st Symposium on the Languages and Logics of Syllogistics (SYLLOS1)

Invited Lectures

The Program of Natural Logic and the Place of the Logic of Names in It

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Tags: logic and artificial intelligence, logic and natural language, names and definite descriptions, syllogistic logic.

This talk has two parts. The first is a very general introduction to the program of "natural logic" [1, 2, 6]. The overall idea is to do logic in natural language as much as possible, ideally without translation into standard logical systems like first-order logic. One of the reasons for pursuing this program is that we connect to classical syllogistic logic, and extend it. This is relevant to Symposium on the Languages and Logics of Syllogistics. For this, see [3, 4]. The second part of the talk is especially about the logic of names, connecting to natural language semantics. I will outline the logical systems in [5] and also mention the opportunities and challenges of working on names and definite descriptions in the context of natural logic.

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On De Re Modal Logic of Names: Semantics and Axiomatisation

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Tags: calculi of names, modal logic of names.

In the semantic study of non-modal and modal logics of names, from a formal point of view, instead of talking of substitutions of general names for name letters, it is better to use set-theoretic semantics.

The prevailing view on Aristotle's modal syllogistic is similar to that expressed by R. Smith (p. 45 in: "Logic", pages 27–65 in J. Barnes (eds.), *The Cambridge Companion to Aristotle*):

> In recent years, interpreters have expended enormous energy in efforts to find some interpretation of the modal syllogistic that is consistent and nevertheless preserves all (or nearly all) of Aristotle's results; generally, the outcomes of such attempts have been disappointing. I believe this simply confirms that Aristotle's system is incoherent and that no amount of tinkering can rescue it.

However, we will try to construct in a relatively natural way axiomatic systems of de re modal logic of names that will, to some extent, agree with Aristotle's modal syllogistic. For these systems, we will present quite natural semantics adequate to them.

Natural Language Inference: From Aristotle to AI

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Tags: logic and artificial intelligence, logic and natural language.

For most of recorded history, logic amounted to little more than a series of sporadic attempts to systematize entailment patterns observed in natural—that is to say, human—languages. Indeed, it was not until the development of the conceptual idiom of the logical variable at the end of the nineteenth century that logic broke these anthropocentric shackles, and blossomed into the abstract theory we know today. Recently, however, a reverse trend has been evident. For it is the very particularity of the logical patterns found in natural language that has increasingly caught the eye of both logicians and computer scientists. This is partly for practical, and partly for theoretical, reasons. Specifically, the realization that with increased expressive power comes increased computational complexity has thrown the spotlight back on logical systems owing their salience to the syntax of natural, rather than, formal, languages. In this talk, I will survey some recent trends in the development of logics for natural language, and report on recent attempts to teach them to large language models.

1st Symposium on the Languages and Logics of Syllogistics (SYLLOS1)

Contributed Lectures

Following From and Being a Conclusion Of in Aristotelian Consequence

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Tags: history of logic, non-classical logics, philosophical logic, syllogistic.

This talk explores a possible divergence between Aristotle's account of syllogistic inference and modern accounts: while the latter treat logical consequence as a relation that may obtain between any combination of premises and conclusions, Aristotle's account imposes structural constraints on what qualifies as an inference in the first place—whether valid or not. Building on [1], I argue that there is a distinction implicit in Aristotle between what *follows from* a set of premises and what can *be a conclusion of* them. The former refers to what can be said to follow logically from a set of premises, as understood within some modern logical framework. The latter, however, requires that the inference conform to an accepted inferential structure—one which, for Aristotle, typically involves at least two premises and excludes cases of circular reasoning. This distinction may help explain why inferences such as $A \therefore A$, or those involving contradictory premises (cf. [2, 3]), though valid by modern standards, do not qualify as syllogisms in Aristotle's system. I therefore propose that Aristotle's notion of logical consequence encompasses not only the requirement that the conclusion follows necessarily from the premises, but also that the inference instantiate a legitimate argumentative form. On the basis of this distinction, I will outline a framework for Aristotelian-style systems that accommodate two kinds of logical consequence: one representing logical following, and the other representing that of being a conclusion of a series of premises.

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'Stoicheia' in Posterior Analytics

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Tags: demonstration, elements, syllogistic.

Since the end of the 19th century there has been quite some debate about the term 'stoicheion' (*στοιχεῖον*) in ancient Greek philosophical and mathematical texts.[9] Two relatively recent studies are the ones by T. Crowley ([2]) and M. Malink ([9]). Crowley's paper argues for "the view that Plato is the first to use *stoicheion* in the sense of 'element' " (p. 368). Malink's starting point is a passage in *Physics* 2.3, 195a16-19 (and *Metaphysics* Δ2 1013b17-21), which concerns the notion of 'material cause'. According to [9], "(Aristotle) writes that the hypotheses are material causes of the conclusion" (p. 164) and "Aristotle is usually taken to claim that the premisses of any deduction are material causes of the conclusion" (p. 165). The aim of the author is "to vindicate Aristotle's claim by offering a new interpretation of it" (p. 168). In developing his argument, Malink refers to the meaning of the term *στοιχεῖον* in Book I of *Posterior Analytics*, and contends, together with others before him, that "Aristotle's remark that there are as many elements as middle terms ..... is not entirely correct" (p. 180).

In this talk, we will present an interpretation of the term 'stoicheion' in this very passage, which is compatible with Aristotle's remark about the number of elements. Our main thesis is that the term 'elements' (*στοιχεῖα*) in the same passage refers to immediate propositions that are (a) universal and (b) necessary, as hypotheses, in the proof procedure required for arriving at a specific conclusion.

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Boethius, “Boethius’ Thesis”, and “Converse Boethius’ Thesis”

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Tags: Boethius, Boethius’ Thesis, connexive logic.

In the wake of McCall’s investigation [5], so-called “Boethius’ thesis” is usually understood as the claim that the implications ‘If p then q’ and ‘If p then not-q’ can’t be together true:

BT $(p \to q) \to \neg(p \to \neg q)$.

The main source for ascribing BT to Boethius is the oft-quoted passage from *De Syllogismo Hypothetico*: “Si est A, cum sit B, est C; [...] atqui cum sit B, non est C, non est igitur A” [2, 851C](Boethius (1847), 851C). As the context of Boethius’ works makes clear, ‘A’, ‘B’, and ‘C’ stand for terms such as ‘homo’, ‘animal’, ‘rationale’, etc. However, according to McCall, the idea behind Boethius’ dictum can be formalized by BT. Recently Wansing and Omori [7] claimed to have found evidence for assuming that Boethius also endorsed the “Converse Boethius’ Thesis”

CBT $\neg(p \to q) \to (p \to \neg q)$.

However, there are strong reasons to suppose that this is not the case. The basic intuitions which led Boethius to claim that two “contrary” implications can’t be together true, clearly show that these implications can be together false. E.g. for Boethius, ‘Si est animal *(A)*, est homo *(B)*’ is false because something’s being an animal doesn’t entail or “include” being a man. But ‘Si est A, non est B’ is equally false because being an animal does not exclude being a man.

According to Dürr [3], Berka [1], Martin [4], and Speca [6], the main evidence for an endorsement of CBT was put forward in Boethius’ commentary on Cicero’s *Topica*. Besides this work, also all relevant parts of *De Syllogismo Hypothetico* shall be scrutinized. It will turn out that Boethius explicitly declared BT to be valid, but he didn’t claim to have invented this principle; he rather attributed it to Aristotle. Moreover, the inferences which apparently conform to CBT are assumed by Boethius to hold only under certain presuppositions concerning the terms A, B, C as they occur in propositions p, q. Somewhat incomprehensibly, these presuppositions (as well as Boethius’s general conception of conditionals) have been ignored or overlooked by other scholars so far.

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Logemes as a New Approach in Studying Ancient Logics

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Tags: history of logic, philosophical logic.

It is worth noting that from the point of view of pure mathematics, all these logics: Aristotelian, Stoic, Epicurean, Buddhist, etc., are not logical theories in the narrow sense. They are not axiomatized (with the exception of Aristotelian syllogistics), do not have algebraic structures, and therefore cannot be represented in the form of mathematical logic. But at the same time they contain the doctrine of drawing conclusions based on a certain set of inference rules.

Let us try to evaluate this science from the point of view of pure mathematics. Let us take the logical theory T as understood in mathematical logic. Now we can take a fragment F from T. If this fragment can form reasoning, it is called a *logeme*. Therefore, assume that the set of formulas F' will be consistent and assume that at least one inference rule can be applied in F', then F' is a logeme.

We say that the logeme F has meaning if and only if its Lindenbaum–Tarski algebra is a poset. Homotopy types allow us to identify different logemes. Let F and F' be two logemes. They are considered identical if and only if their posets are of the same homotopy type. This approach can be helpful in studying ancient logics based on their posets by topological tools. For example, the Stoic logeme simplicially collapses to a point.

Barbara, Celarent...: The Epistemology of Syllogistic Mnemonics

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Tags: history of logic, philosophy of logic, philosophy of science.

Syllogistic mnemonic devices were widely used as tools for remembering various properties of the system of Aristotelian assertoric syllogistic throughout the Middle Ages and beyond. The pinnacle of their development and simultaneously the most important example of syllogistic mnemonic is the XIII-century short poetic piece beginning with *Barbara, Celarent...*, which encodes and systematises information pertaining to the structural aspects of syllogistic moods and the methods of proving their validity [1]. My talk, beginning with a short introduction into the workings and history of the *Barbara, Celarent...* mnemonic, offers a glimpse into how the structural aspects of this mnemonic device itself have determined the subsequent development of assertoric syllogistic after XIII century.

To this end, I propose to look not only at the changes made by scholars *to* the *Barbara, Celarent...* mnemonic, but at the development of new syllogistic mnemonic names going *beyond* the boundaries of this mnemonic. I draw attention to two of such developments—the first one by Jean Buridan, pertaining to finding new indirect syllogistic moods [2, p. 331–334], the other by G. W. Leibniz, consisting of finding new subalternated moods [3, p. 458–479; 486–490]—and argue that while these developments undoubtedly reflect the general trend of noticing indirect and subalternated moods, they are themselves constrained by what the previously developed mnemonic system allows to be spotted and thought of. The talk shall therefore both offer a historical perspective on the development of syllogistic logic as seen through the lenses of the *Barbara, Celarent...* mnemonic and provide some comments on the epistemological significance of this—and similar—mnemonic systems.

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3rd Workshop on Non-Fregean Logics (WNFL3)

Invited Lectures

Simplicial Epistemic Semantics

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Tags: epistemic logic, many-valued logic, modal logic, semantics of modal logic.

All my working life as a logician epistemic logic came with Kripke models, in particular the kind for multiple agents with equivalence relations to interpret knowledge. Sure enough, I knew about enriched Kripke models, like subset spaces, or with topologies. But at some level of abstraction you get back your standard Kripke model. Imagine my surprise, around 2018, that there is an entirely dual sort of structure on which the epistemic logical language can be interpreted and that results in the same S5 logic: simplicial complexes. Instead of points that are worlds and links labeled with agents, we now have points that are agents and links labeled with worlds. Or, instead of edges (links), triangles, tetrahedrons, etcetera, that represent worlds. Simplicial complexes are well-known within combinatorial topology and have wide usage in distributed systems to model (a)synchronous computation. The link with epistemic modal logic is recent, spreading out from Mexico City and Paris to other parts of the world, like Bern, Prague and Vienna. See [5, 8, 9]. Other logics are relevant too, for example KB4, in order to encode crashed processes/agents. See [3, 6] Other epistemics notions are relevant too: distributed knowledge facilitates further generalizations from simplicial complexes to simplicial sets, belief instead of knowledge facilitates generalizing to oriented simplicial complexes. See [2, 4, 8]. Three-valued logics are relevant too, in order to represent crashed processes and dead agents. See [1, 7, 9]. It will be my pleasure to present my infatuation with this novel development connecting epistemic logic and distributed computing.

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Beyond Truth Values:
A Non-Fregean Approach to Logical Relevance

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Tags: algebraic logic, non-classical logic, non-Fregean logic, relevant logic.

The notion of *non-Fregean logic*, introduced by Suszko in [2], is most commonly associated with SCI (*Sentential Calculus with Identity*) and its extensions – systems that extend classical propositional logic with a new connective $\equiv$, intended to represent a congruence on the domain of semantic correlates of sentences. According to Suszko, a genuinely non-Fregean framework must reject Frege's Principle, which identifies the meaning of a sentence with its truth value [3]. As a consequence, the so-called Frege Axiom

(AF) $\quad (\varphi \leftrightarrow \psi) \to (\varphi \equiv \psi)$

cannot be a valid formula in any non-Fregean logic. Suszko regarded (AF) as a formal representation of Frege's Principle, and accordingly classified any logic validating (AF) as *Fregean*.

However, as argued in [1], the validity of (AF) as a criterion for Fregeanity is philosophically and technically inadequate. In particular, it may fail when any of the connectives involved in (AF) is not explicitly present in the language of the logic under consideration and/or when the connectives $\leftrightarrow$ and $\to$ are interpreted non-classically. Instead, the paper [1] proposes a general non-Fregean framework, which allows for a systematic comparison of logics that differ in language or semantics, or both. Within this framework, three increasingly strong notions of a *non-Fregean logic* have been defined: weakly non-Fregean, non-Fregean, and standard non-Fregean. Each of these corresponds to a class of logics that meets the most fundamental philosophical requirements underlying Suszko's non-Fregean methodology.

The results presented in [1] have shown that these three classes cover not only SCI and its classical extensions but also a broad range of non-classical systems, including non-standard non-Fregean logics, Grzegorczyk's logics of descriptions, as well as modal, many-valued, intuitionistic, paraconsistent, and relevant logics.

In this talk, we will discuss properties of relevant logics from the perspective of non-Fregean methodology. In particular, we will show that the relevant logic R is a standard non-Fregean logic. The paper will also examine metalogical inferential relationships between relevant and other non-Fregean logics. The logics under consideration will be compared with respect to both their sets of valid formulas and their semantic consequence relations.

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3rd Workshop on Non-Fregean Logics (WNFL3)

Contributed Lectures

Fregean Definite Descriptions in Non-Fregean Logic

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Tags: description theories, non-classical logic, proof theory.

The talk is devoted to developing a Fregean theory of propositional definite descriptions in the context of non-Fregean logic, that is propositional logic with new connective—propositional identity.

Definite descriptions are typically regarded as first-order term-forming expressions that denote uniquely determined objects. Given an individual variable x and a formula φ, the expression $\iota x \varphi$ constitutes a term. Definite descriptions are commonly classified as either proper or improper. Proper descriptions have a unique referent, while improper ones lack such a referent.

A distinctive feature of the Fregean theory of definite descriptions [1, 2] is the assignment of a specially designated object as the referent of all improper descriptions. In contrast to Russell's reductionist approach [4], which seeks to eliminate descriptions by translating them into standard first-order logic with identity, the Fregean perspective treats all descriptions as genuine terms.

We propose an approach rooted in Suszko's non-Fregean logic [5], working with language having both propositional quantification and propositional definite descriptions. Given a propositional variable p and a formula φ, the expression $\iota p \varphi$ is a formula denoting the unique proposition that satisfies φ. We designate a special propositional variable as the referent of improper propositional descriptions instead of specially designated object. Propositional identity connective $\equiv$ plays a central role in the development of this framework.

We aim to provide a semantic account of this approach and to modify Indrzejczak's [3] cut-free sequent calculus for Fregean theory to accommodate propositional definite descriptions.

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Around S0.5, SCI and WB: New Bridges Between Modal and Non-Fregean Logics

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Tags: algebraic logic, modal logic, proof theory, semantics of modal logic.

Non-Fregean logics constitute a formal explication of the rejection of the so-called *Fregean Axiom* (FA) [8], according to which sentences denote their truth-values. By contrast, non-Fregean logics, formally obtained by adding identity connective $\equiv$ to the classical propositional logic, allow us to differentiate between a sentence's denotation and its truth value.

The subject of the presented research is two non-Fregean logics, the weakest among those introduced by R. Suszko ([6, 7], cf. [1]): the Sentential Calculus with Identity (**SCI**) and its Boolean extension **WB**. **SCI** characterizes the connective $\equiv$ as a congruence, and **WB** extends **SCI** with Boolean axioms expressed by means of $\equiv$. Already in [7], R. Suszko demonstrated that the logics **WT** and **WH**, further extensions of logic **WB**, are notational variants of modal logics **S4** and **S5**, respectively. On the other hand, T. Ishii [2, 3] showed that other well-known normal modal logics are modal counterparts of some non-Fregean logics with identity. For this purpose, he modified **SCI**, introducing the logic **PCI** and its axiomatic extensions. However, modal counterparts of **SCI** and **WB** have not been established.

In our talk, we start with modal logic **S0.5** [4, 5] and modify its semantics in order to obtain Kripke-style semantics for logics **SCI** and **WB**. We also introduce two new logics: **WB0.5**, which brings us closer to the non-Fregean counterpart of **S0.5**, and $\mathbf{S0.5_{con}}$, which brings us closer to the modal counterpart of **WB**. The following results will be discussed: algebraic semantics for **WB0.5**, based on a modified version of algebraic semantics for **SCI** and **WB**; Kripke-style semantics for **SCI** and for $\mathbf{S0.5_{con}}$; and tableau systems for **SCI** and **WB** based on the Kripke-style semantics.

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Tutorial: Formal Theories of Definite Descriptions (FTDD)

Formal Theories of Definite Descriptions. Part I

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Tags: history of logic, non-classical logic, philosophical logic, proof theory.

A definite description is an expression of the form 'the F': the first man on the moon, the author of *Principia Mathematica*, the present King of France. It purports to refer to the unique object that answers to the description. If it succeeds, the definite description is proper, if not, improper. Presumably everyone agrees about the proper definite descriptions. The question is what to do with the improper ones.

The present tutorial presents a number of approaches to answering this question and formalisations of the ensuing theories of definite descriptions. (For an overview, see [7].)

Frege saw the importance of definite descriptions in mathematics and his *Basic Laws of Arithmetic* [2] contains the first axiomatisation of a theory of definite descriptions. It is rather unusual and idiosyncratic. The theory he is most known for is the so-called chosen object theory [1], also advocated by Carnap [3]. Improper definite descriptions refer to a more or less arbitrarily chosen object. The truth conditions of sentences containing improper definite descriptions may be unexpected, but are inoffensive. For instance, choosing, as Frege proposes, the number 0 as the referent of all improper definite descriptions, it turns out that 1 is the successor of the present King of France. The point is that the theory gets the truth conditions of sentences containing proper definite descriptions right, and we care little about the others.

Ever since Russell's celebrated Theory of Definite Descriptions [4] (see also [5] and [6]), these expressions have taken centre stage in logic and philosophy of language. Russell argued that, despite appearance, definite descriptions are not, in fact, singular terms. They only have meaning in the context of complete sentences 'The F is G', and these mean 'There is exactly one F and it is G'. The definite description disappears upon analysis. If there is no F or more than one, it is false. Russell's theory can be elegantly formalised by appealing to an idea of Arthur Prior's. Complete sentences 'The F is G' are formalised by a binary quantifier I as $Ix(F, G)$. This approach has also been defended by Neale. Rules of inference in natural deduction and sequent calculus for I are straightforward.

Russell's theory, albeit considered by many to be a paradigm of philosophy, is not without critics. In particular, it is felt that definite descriptions really are singular terms and should be treated as such. An aim of the development of free logic by Hintikka and Lambert was to formulate theories of definite descriptions that are alternatives to Russell's ([8], [9]). Free logic avoids existence assumptions of classical logic, namely, that the domain of quantification must be non-empty and that terms must denote (existing objects). Quantifiers, however, retain their existential import. Consequently, quantifier rules must be restricted to terms that denote (existing objects). Concerning definite descriptions, a guiding thought was that logic should remain largely silent on the improper ones. Definite descriptions are governed by what has come to be called Lambert's Law, and this only says that something is the F iff it is the unique F. Lambert's Law is usually formulated as an axiom; here we'll present rules for natural deduction and sequent calculus. The chosen object theory is an extension of the ensuing system. Finally, the binary quantifier approach can also be applied fruitfully in free logic.

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Formal Theories of Definite Descriptions. Part II

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Tags: modal logic, non-classical logic, proof theory, temporal logic.

In this part of the tutorial on the theory of definite descriptions (DDs), I will show how Russell's theory of DDs can be incorporated into the modal and temporal setting. In particular, I will introduce two logics: propositional modal logic with DDs, named $\mathcal{ML}(\mathsf{DD})$, and first-order hybrid temporal logic with two types of DDs, referred to as $\mathsf{FOHL}_{\iota,\lambda}$.

In the first logic, DDs take the form $@_{\varphi}\psi$, which can be interpreted as 'the (unique) modal world which satisfies φ, satisfies ψ, too. I will show that adding such operators to the basic (propositional) modal language has a price of increasing the complexity of the satisfiability problem from PSpace to ExpTime. However, if formulas corresponding to descriptions are Boolean only, there is no increase of complexity. I will present a comparison of DDs with the related operators from hybrid and counting logics and sketch a proof of the fact that DDs in $\mathcal{ML}(\mathsf{DD})$ are strictly more expressive than hybrid operators, but strictly less expressive than counting operators. I will also demonstrate how these expressiveness relations fluctuate over different classes of structures and how the bisimulation notion needs to be adjusted to adequately capture the notion of modal equivalence in $\mathcal{ML}(\mathsf{DD})$.

In $\mathsf{FOHL}_{\iota,\lambda}$, DDs occur as the only non-rigid terms. I will introduce the logic as formalised by means of a tableau calculus working on sat-formulas. A particular theory of object DDs exploited here is essentially based on the approach of Russell, but with descriptions treated as genuine terms. However, the reductionist aspect of the Russellian approach is retained in several ways. Alongside standard (that is, referring to objects) DDs, the language of $\mathsf{FOHL}_{\iota,\lambda}$ comprises another type of DDs that refer to time instances (and characterise them by virtue of events occurring exclusively at these time instances). I will show that if the hybrid machinery involved in the logic is full-fledged, i.e., encompasses state variables and binders, the Craig interpolation property can be proved for the logic in a constructive manner using the previously introduced tableau system.

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Formal Theories of Definite Descriptions. Part III

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Tags: proof theory, set theory and the foundations of mathematics.

The methodology of developing proof systems for definite descriptions as formalised by means of iota operator can be applied also to other term-forming operators. In the last part of the tutorial we will show how it can be applied to abstraction operator in set theory. Instead of providing a system adequate for specific theory of sets, like ZFC or NBG, we will focus on the formalisation of some weaker, more basic theory called virtual theory of classes (VTC), which was introduced by Quine [1] as a flexible tool for comparison of different approaches to set theory. Quine did not provide an axiomatization of VTC but it was done by Scott [2] who specified also a model-theoretic characterization and completeness proof, but using positive free logic instead of classical logic used by Quine.

Since VTC specifies in a non-committal way the conditions for reasoning with set theoretic constructions, it may be seen as a kind of the logic of classes without existential assumptions. Recently, also Tennant [3] provided a variant of logic of classes in the form of natural deduction calculus, but based on the negative free logic (NFL). Moreover, he emphasized that such an approach may be seen as a modified, weaker form of logicism, called structural neologicism. In fact, logicism in its strong form, reducing mathematics to logic, is rather difficult to defend, however, it may be claimed that mathematisation of set theory was premature. The variety of axiomatic set theories, often incompatible, leads to the question if some basic neutral theory of sets can be provided. What we need is the logic of classes dealing with the concept of class (characterised by arbitrary formulae), rather than with the existential commitments of particular axiomatically formulated theory of sets (the existing classes). Accordingly, the rules of such logic should be existentially non-committal but characterising deductively our notions as fully as possible. This way a firm distinction between the meaning and the existence of mathematical objects is preserved.

VTC is arguably a simpler logic of classes than Tennant's or Scott's proposals. It has also a potential to be used as a practical tool for automated deduction in set theory, but it was not explored proof-theoretically so far. We present a sequent calculus GVTC which adequately characterizes VTC. It is based on the well-behaved rules characterising set-abstracts, and allows for constructive proof of cut elimination and consistency of VTC. The commonly applied definable notions of set theory can be also introduced in a modular way by means of well-behaved rules preserving cut elimination. Still, the enriched variants do not allow for the proof of the existence of suitable sets (although one can prove explicitly the nonexistence of several paradoxical classes).

It is also possible to enrich GVTC with existential axiomatic sequents (or corresponding rules) to obtain a calculus adequate for set theories like ZFC or NGB. One strategy for doing this will be briefly discussed. This shows also the usefulness of GVTC in proof search, and possibly, automation of proofs in set theory.

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Author Index

X

Y

Z